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THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS (TEORIY--ETC(U)

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# FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS

by

G. I. Barenblatt, V. M. Yentov, V. M. Ryzhik

FOREIGN TECHNOLOGY DIVISION



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Table of Contents

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Page 146.

Chapter VI.

THE FILTRATION OF INHOMOGENEOUS LIQUIDS.

1. the filtration of polyphase liquids.

1. In connection with design and analysis of the development of petroleum and gas fields, is necessary to examine consistent motion in simple medium of several liquids, most frequently the water, oil and the gas, which are the isolated phases, which are not mixed between themselves.

In order to describe the filtration of polyphase liquid in connection with the model of continuous porous medium, it is necessary to introduce the characteristics of the averaged motion. The scale of averaging in this case must be great not only in comparison with the significant dimension of pores, but also with the size of particles of each of the phases. It is substantial, that the minimum size of particles can considerably exceed the size/dimension of pore channel; therefore the scale of the averaging of is determined by the character of phase distribution in pores can be various depending on the formulation of the problem. In more detail about this it will be said below, thus far let us assume that there is this linear dimension r, with which the characteristics of each of the phases, averaged on sphere with a radius of R, have with $R \to r$ asymptotic extreme values, but size/dimension r are considerably less than the characteristic scales of the problems in question. The main

characteristics of the filtration of polyphase liquid are the saturation and the rate of filtration of each phase.

The volume ratio of pores in elementary macrovolume in the vicinity of this point, occupied by the i-th phase is called the local saturation of pore space by this phase and is designated aaaa. It is obvious,

$$\sum_{i=1}^{n} s_i = 1, \qquad \text{(VI.1.1)}$$

where n is a number of independent phases. Thus, in system n of phases is by n - 1 independent saturation. Specifically, during the study of the filtration of two-phase liquid it suffices to examine only one saturation.

Page 147.

The motion of yeachyu of the phases can be described by the velocity vector of the filtration of the given phase of $\vec{u}_i$. Analogous with

the rate of filtration of single-phase liquid, $\vec{u}_i$ is defined as vector whose projection on certain direction is equal to the volumetric flow of this phase through the single area/site, perpendicular to the indicated direction.

The characteristic features of the motion of polyphase liquid are connected with the effect of surface tension. As is known, on the bent boundary of two phases, appears the pressure shock, equal to

$$p_c = \alpha\left(\frac{1}{R_1}+\frac{1}{R_2}\right), \qquad \text{(VI.1.2)}$$

(Laplace's formula), where $\alpha$ are interphase tension, $R_1$ and $R_2$ are the main radii of curvature on the surface of the section of phases at the particular point. In the porous medium the boundary of two phases is divide/marked off into many individual sections. The radius of curvature of each of them is close by order of value to the size/dimension of pore channel [1].

FOOTNOTE [1]. We clear the case when one of the phases is found into

another in the form of the emulsion, radius of bubbles of which it is considerably less than the size/dimension of pore channels. In this case liquid can be considered as quasi-homogeneous; the theory of the filtration of such emulsions for the case of system gas - liquid was developed by L. S. Leybenzon [73]. ENDFOOTNOTE.

Capillary pressure shock, especially in low-penetrable media, can play the significant role in the process of filtration. For example with the permeability of the porous medium of approximately 10 md a radius of the pores of sandstone comprises approximately $10^{-4}$ cm. and capillary pressure on boundary gas - water it is of the order 0.5 kgf/cm².

The effect of capillary forces on filtration processes pronounces in a two-fold manner. The motion each of the phases polyphase system depends on the forces of pressure, which cause motion, and on the relative location of phases in pore space. The phase distribution in pores lays out of the region of the flow of each of the phases and thereby the value of the resistance, experience/tested by this phase during motion, since the structures of pore space determines hydraulic resistance during single-phase current. Capillary forces affect both the distribution of pressure in

phases and on the relative location of phases in pore space. Respectively the processes of the filtration of polyphase liquid go differently depending on characteristic time of filtration process and on the size/dimensions of zone of flow. Capillary forces create in the porous medium a jump/drop in the pressure whose value is limited and does not depend on the size/dimension of region. The external pressure differential, which creates the filtration flow between two points, is proportional to rate of filtration and to dista[illegible]tween these, is proportional to rate of filtration and to dis[illegible]between these points. If the size/dimensions of region are small, then during sufficiently slow movement capillary forces can exceed an external pressure differential. Therefore in this region the set-up time of the equilibrium distribution of phases is considerably less than time in which occurs a noticeable change in the average/mean saturation under the action of filtration flow.

Page 148.

Thus, during the study of local processes, i.e., the processes, which proceed in elementary macrovolume, phase distribution in pores usually can be considered equilibrium. This approach let us assume if saturation is not changed by noticeable form at the distances of the

order of the size/dimension of pore channels. In other words, the dimensionless quantity of $\sqrt{k}|\mathrm{grad}\, s|$ must be small. On the contrary, if is examined motion in very lare region (for example in the whole petroleum deposit), then to the effect of capillary forces on the distribution of pressure insignificantly and their action pronounces indirectly, through the local processes of the redistribution of phases, which produce change in the local hydraulic resistance. Finally, in series of problems it is necessary to examine flow in the regions of such intermediate size/dimensions, that the duration of the processes of redistribution caused by capillary forces, is comparable with characteristic at times filtration. The problems of this type are encountered, for example, during the study of the processes of the displacement of oil or gas by water from heterogeneous or fissured rocks.

Since us interest the local characteristics of large-scale motion, we will examine the equilibrium distribution of phases, without investigating the process of its establishment. However, even, equilibrium phase distribution with the same degree of saturation can be different. Although the phase distribution does not depend on the average rate of filtration, it depends substantially on how this saturation originated.

In a number of cases, one or several phases can be found in pores in the form of the isolated/insulated bubbles or drops, not connected and the remaining part of this phase. Such isolated/insulated bubbles or drops appear either during the isolation of the phase, dissolved in another phase or at the end of the process of the displacement of one phase by another, when the particles of the displaced phase are broken to separate drops. The mobility of the separate drop, surrounded by other phases, in the porous medium is very small and can be equal to zero with those gradients of the external pressure which exist in the basic filtration flow. For an example the typical position of the separate drop of the wetting phase in pores is shown in Fig. VI.1. In order to push this drop through the contraction of pore with a radius of r, it is required to apply the pressure differential of the same order as the excess capillary pressure which it is approximately $\alpha/r$. Therefore at the length of drops in several pore channels motion will be initiated only if pressure gradient will exceed $\sim\alpha/r^2$, that as it is not difficult to calculate, considerably exceeds usual pressure gradients by filtration flows by petroleum and gas layers. Therefore the incoherent part of each phase usually is motionless. Let us note also that the incoherent saturation can be only the small portion/fraction of pore space.

Page 149.

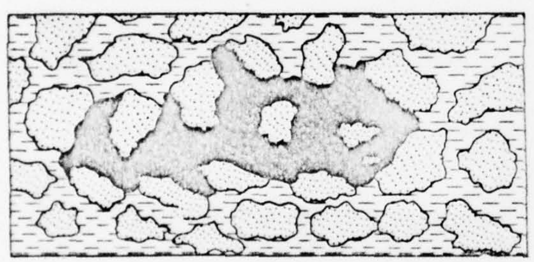


Fig. VI.1.

for the description of the equilibrium distribution of phases in pores is utilized the concept of capillary pressure. In equilibrium, state the phase boundary in pores has the complex branched form. Let us examine two-phase equilibrium in the porous medium. If each phase in the vicinity of certain point of the porous medium is continuous, then possible two hundred concept of the mean pressure in this phase, which in the limit, with the contraction of the surface of averaging can be considered as pressure at the particular point. Thus, at equilibrium of two-phase mixture in the porous medium at each point are determined two pressures: $p_1$ and $p_2$. The difference between these pressures is called capillary pressure m at the particular point of the porous medium. In accordance with Laplace's formula (VI.1.2) the capillary pressure geometric mean curvature of the interphase interfact in the vicinity of this point. Pressure is more from the side of liquid, than the worse wetting the solid phase of skeletal porous medium. Apparently, basic part of the liquid cannot be found in the form of film on the surface of solid phase, but it has boundaries of the type of meniscuses (see work of M. M. Kusakova and L. I. Mekenitskaya [62]). The mean curvature of these meniscuses, obviously, depends first of all on bend pore space, i.e., on the mean radius of the pores of the medium. Further, the curvature of meniscuses and, consequently, also capillary pressure depend on the saturation of pore space by the more wetting phase which has a tendency to occupy more pin-head blisters.

To equilibrium state corresponds the minimum of thermodynamic potential, which is reduced with invariable pressure to the minimum of the surface energy, i.e., the minimum value of the surface area of the section of phases in the given volume. The complex structure of pore space leads to the fact that with this saturation there is a series of the local minimums of surface energy, and the equilibrium distribution is ambiguous. As a result capillary pressure depends at this saturation on the way of saturation. Nevertheless, if we examine the only processes of displacement (without phase transitions), then it is possible to count that the capillary pressure, besides saturation by the wetting phase, depends only on that, increases or decreases saturation for achievement of its assigned value.

Page 150.

If we are distracted from this ambiguity, then of the dimensional considerations it follows that the capillary pressure of $p_c$ in state of equilibrium can be presented in the form:

$$p_c = \frac{2\alpha}{\bar{r}} \psi(s, \Theta), \qquad \text{(VI.1.3)}$$

where the $\bar{r}$ are a characteristic linear size of pores; $\Theta$ - static contact wetting angle; dependence (VI. 1.3) is called capillary curve.

If we examine the porous media whose structures are similar, then as characteristic linear size it is convenient to accept (in accordance with what has been said in chapter I) the value of $\sqrt{K/m}$.

The effect of the angle of the wetting of $\Theta$ on the form of capillary curves experimentally is studied insufficiently. By analogy with the equilibrium of liquid in capillary tube, is accepted to include in formula for a capillary pressure instead of $\alpha$ "tension of wetting", equal to $\alpha$ to cos $\theta$, and to consider that the dependence of aaaa on aaaa by this is contained. Then equality (VI.3.1) is

converted as follows:

$$p_c = \frac{2\alpha\cos\theta}{\sqrt{\frac{k}{m}}} J(s). \qquad (VI.1.4)$$

Representation (VI.1.4) was suggested for the first time by Leverett [143], and the dimensionless function J (s) is called of the function of Leverett. It goes without saying that under actual conditions it cannot exist two media whose microstructures are completely similar. Nevertheless it is experimentally established/installed that function J (s) retains its form with sufficient accuracy for the whole classes of the similar in structure porous media (for example for the separate groups of grains of sand, sands, etc.).

The equilibrium distribution of phases in pore space depends not only on the final saturation, but also on how reached this saturation. To consider entire prehistory of saturation, of course,

is impossible. However, in appendices are most interesting the processes of the gradual substitution of one liquid of another, when change in saturation occurs monotonically. For the analysis of such processes it is possible to use one of the two types the experimental curves of J (s), that are distinguished by the method of their obtaining. If in the investigated process grow/rises the saturation by the less wetting phase, then are utilized the "curved displacements", obtained under conditions, when are the less wetting liquid (or gas) very slowly displaces from the specimen of the porous medium the more wetting liquid. On the contrary, for the processes in which grow/rises the saturation by the more wetting phase, are utilized the "curved impregnations", obtained during the spontaneous capillary displacement of the less wetting liquid (or gas) by the wetting liquid in the vertical column of the porous medium.

Key: (1). Displacement. (2). Impregnation.

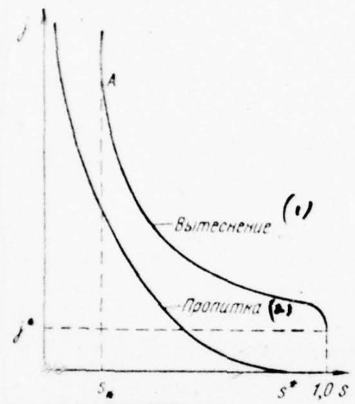



Fig VI.2

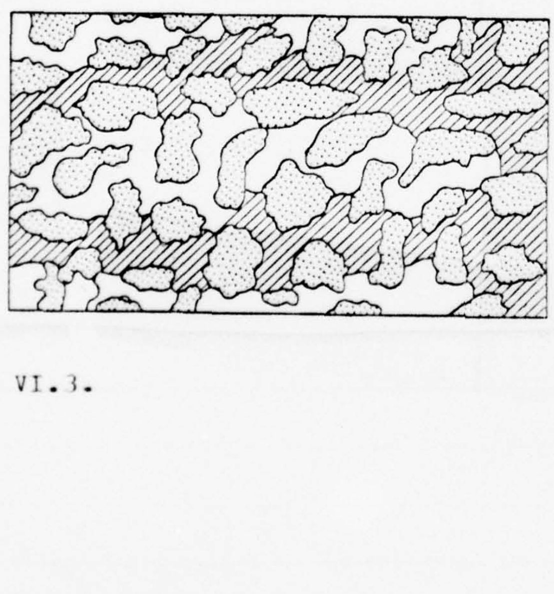


Fig. VI.3.

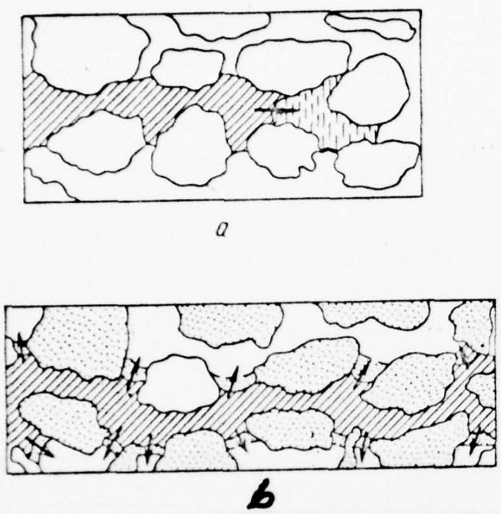
On completion of the penetration of liquid, they are found in hydrostatic equilibrium, and the capillary pressure in each cross section is determined from the formula of the $p_c = \Delta\rho g z,$ where the aaaaaa - a difference in the densities of liquids, z - the height/altitude of this section above the level of the free wetting liquid. By measuring the saturation in each section s, it is possible to construct curved $p_c(s)$ and J (s).

Curved displacements and impregnations noticeably differ from each other (Fig. VI.2), but in practice they do not depend on the properties of the liquids, used for investigation, and each of them can be described by single-valued function saturation.

Let us note still that that part of the liquid, which is found in the form of separate drops (incoherent saturation), is not utilized in the determination of capillary pressure. If entire liquid is found in incoherent state, then the concept of capillary pressure becomes meaningless. Therefore, for example, curved displacements J (s) in Fig. VI.2 actually break themselves with certain $s = s_*$ and do not have a sense with of $s < s_*$.

2. During the filtration of two liquids in the porous medium at least, one of them forms the connected system, which borders to porous skeleton and partially to another liquid (Fig. VI.3). Due to the effect of the selective wetting of solid phase of one of the liquids, the contact area each of the liquid phases with skeleton porous medium considerably exceeds the contact area of the phases among themselves (above it was already mentioned, that existence of one of the phases in the form of film on the surface of solid skeleton is highly improbable).

Fig. VI.4.



This means, that in the first approximation, it is possible to consider that each phase moves in the occupied with it space under the action "of its" pressure independent of other phases, i.e., in the manner that if it was limited only by solid walls. The law of the filtration of each of the liquids of two-phase system can be written in the following form:

$$\vec{u}_i = -\frac{k}{\mu_i} f_i \operatorname{grad} p_i. \qquad \text{(VI.1.5)}$$

the dimensionless quantities of aaaa is conventionally designated as relative permeability. The relative permeability are the most important characteristics of two-phase current [1].

FOOTNOTE [1]. Subsequently of the relationship/ratio of form (VI.1.5) will be applied to problems the displacements of one liquid of another. It would seem, here there is a direct/straight contradiction with assumption about the independent variable of the motions of each

of the phases. However, if saturation in the course of displacement are changed not too rapidly, then the process of displacement will occur in essence not by the "expulsion" of particles of the displaced liquid in direction of motion (Fig. VI.4a), along it is faster by the path of gradual "forcing back" it to the side, perpendicular to movement (Fig. IV.4b). ENDFOOTNOTE.

If we do not consider inertia effects, then each of the function aaaa depends only on the dimensionless parameters s, $k\,|\text{grad } p|\ /\ \alpha$, $\mu_1/\mu_2$, m.

The large experimental material, accumulated at the present time [127], shows that the form of the curves of relative permeability for this porous medium barely depends on the nature of liquids and is determined mainly by the preferred wettability.

Page 153.

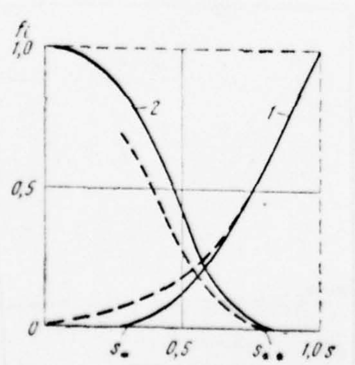



Fig. VI.5.

Specifically, from the majority of the experimental data it follows that the relative permeability cn depend on the relation of the ductility/toughness/viscosities of phases. This serves as the confirmation of the above-made assumpticn about the fact that the flow of each liquid in the occupied with it space occurs independent of the motion of adjacent phase [1].

FOOTNOTE [1]. The experimental results of oppcsite character are contained in work to ode [150]. However, experiments to ode are carried out with somewhat artificial conditions and are related to the case when the low-viscosity wetting phase forms on the surface of solid skeleton the continuous boggy layer, which plays the role of lubrication. Under these facts the relaticn of ductility/toughness/viscosities enters in the number of determining parameters. ENDFOOTNOTE.

The parameter of $\pi_\alpha = k|\text{grad}\, p|/\alpha$ characterizes the ratio in the external pressure differential at the distances of the order of the size/dimension of pore channels to capillary pressure. Above it was noted that for usual sufficient slow movements this relation is small, and the gradient of external pressure cannot substantially influence the phase distribution in pores. Therefore under these

conditions relative permeability can be considered the functions only of saturation.

If we consider that the resistance to motion of each phase is determined only by structure occupied by it parts of the pore space, then the value of $\kappa f_i$ is for this phase of penetrability (sometimes $\kappa f_i$ are called phase permeability) in usual understanding. It is obvious that with an increase of saturation by this phase from zero to one the function of $f_i$ also grow/rises from zero to one. Above it was shown that the phase distribution in pores depends actually only on which of the liquids possesses the preferred wettability, but not from the individual properties of liquids. Hence it follows that the form of the function of $f_i(s)$ since and J (s) is determined by the structure of pore space.

From the fact that the more wetting phase occupies with the same saturation, the finer the porosity, the less wetting, it follows that and relative permeability with the same saturation for the less wetting liquid is more than for more wetting. Therefore, the curves of the dependences of relative permeability on saturation as one of the phases (for example more wetting) take usually the asymmetric form of the type, depicted on Fig. VI.5.

It is characteristic that for each phase there is a maximum saturation ($s_*$ and $1-s_{**}$) - such, that with less saturation this phase is immobile, i.e., is found in incoherent state (index 1 in Fig. VI.5 is related to the more wetting liquid, index 2 - to less wetting).

Page 154.

With an increase in the rate of filtration by phase distribution in pores, begins to affect the gradient of external pressure. According to D. A. Efros's data [127], this effect begins to show up in relative permeability in the parameter of $\pi_\alpha = k|\mathrm{grad}\, p|/\alpha$, greater than $10^{-5}$ (т. е. $\pi_\alpha' = r^2|\mathrm{grad}\, p|/\alpha \approx 10^{-3}$.

The application/use of equilibrium conditions to the problems of the filtration of two-phase liquid considerably complicated in view of the nonuniqueness of phase distribution during that which was assigned saturation. Supplementary complications appear as a result

of the wetting hysteresis, capillary hysteresis, change in the properties of solid skeleton under the action of prolonged contact with liquid and another analogous physicochemical phenomena. Therefore the form of the curves of relative permeability and curves J (s) (see Fig. VI.2), it depends on that, is raised either is reduced saturation in this process, or in the more general case - from an entire prehistory of process. However, for the most virtually interesting processes in which the saturation vary monotonically, relative permeability, exactly as and function J (s), can be considered as single-valued functions saturation.

The curves of relative permeability and J (s), are divided using the method of their obtaining into "curved displacements" and the "curved impregnations", which can noticeably be distinguished (see Fig. VI.5 - "curved impregnations" are noted by dotted line).

The region of incoherent saturation (s < s1* in Fig. VI.5) also depends on the way of the saturation of the porous medium and in the process of displacement is different depending on whether grow/rises or decreases the saturation by this phase. If this phase is displacing, then motionless (incoherent) can be the only that part of this phase which initially saturated the porous medium. For example

in the case of the displacement of oil by water the displacing water is completely movable, but the water, which was being located is earlier in layer with small initial water saturation (connate water), can be motionless in that part of the layer, where did not reach the forced water. This is explained by the fact that the buried water is located in layer in the form of separate drops and becomes movable, only uniting with the base mass of intruding into layer water.

The most essential deflection of curved relative permeability from the equilibrium, related both with different hysteresis phenomena and with the discontinuity of phases, it is noted during the filtration of the gassed liquid. The distribution of gas and liquid in pore space with the same saturation is substantial differently in cases when gas appeared in pores "from without", displacing liquid, and when dissolved previously gas was isolated from liquid. The gas, which came "from without", moves as the continuous phase, which is in capillary equilibrium with liquid. For gas in this state, the values of relative permeability depend only on saturation and coincide with usual relative penetrabilities for the nonwetting phase. At the same time the gas, which isolated from solution/opening, is located in pores in the form of separate bubbles. As noted D. A. Efros's experiments [127], relative permeability for gas in this state with the same saturation many

times less than for continuous gas phase. Deviation from capillary equilibrium is exhibited also in the fact that the relative permeability this case depend on the relation of ductility/toughness/viscosities. Relative permeability for the gas, which isolated from solution/opening, in larger degree, than usual relative permeability, depend on the prehistory of process the liberation of gases and its subsequent motion, since these processes affect value and amount of bubbles. / D. A. Efros showed that as first approximation it is possible to consider relative permeability depending on saturation and on reduced pressure, i.e., on the ratio of pressure in gas to saturation pressure of petroleum dissolved by gas.

Pages 155-184.

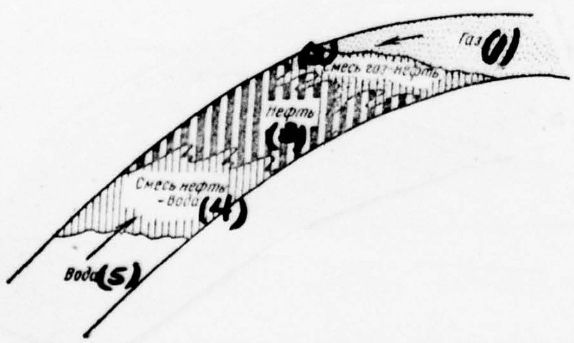



Fig. VI.6.

Key: (1). Gas. (2). Mixture is gas- oil. (3). Oil. (4). Mixture is cil-water. (5). Water.

§2. Displacement of the nonmiscible liquids. Prcblem of Bakley-Leberett.

The extraction of oil from layer in the majority of cases takes a course of the displacement of it by water or gas. The skeletal diagram of this process is depicted on Fig. VI.6. Water either enters the oil-saturated part of the layer, moving from the water-saturated zones of the same layer (limb zone), or artificially it is pumped into into the layer through special injecticn wells. The same displacing agent can be the gas, which is located in layer or specially forced. Oil is extracted through operational holes, whereupon in a number of cases together with oil is extracted water or gas, that burst open to operational holes.

Page 156.

Although the usually natural gas is take/selected from layer because of its expansion during a decompression, in certain cases and gas

layers work under the conditions of the displacement of gas by the water, which saturates the exteriors of the same layer. For the analysis of the effectiveness of the displacement of oil or gas by the nonmiscible with it liquid it is necessary to know, as changes the distribution of saturation in layer. In connection with this the displacement of the nonmiscible liquids must be considered as process of two-phase filtration.

Let us derive the system of equations of two-phase filtration in the homogeneous porous medium without phase transitions. During the notation of the law of filtration, we will assume that at any point each of the phases is located by pillar only in one of the two end states: a) connected and movable and b) incoherent and therefore motionless. In accordance with what has been said in the preceding/previous paragraph, in the flow regions of slow thermodynamically equilibrium flow for those liquids which it is possible to consider connected, it is possible to introduce into the calculation the depending only on saturation relative permeability of $f_i(s)$ and the capillary pressure of $p_c(s)$.

Furthermore, clear hysteresis effects, we will examine the only unidirectional processes which are the most interesting for

application/appendices.

As a result of the made assumptions the law of filtration can be written in the form:

$$\bar{u}_i = -\frac{k}{\mu_i} f_i(s)\,\mathrm{grad}\, p_i \qquad (i = 1,\ 2). \qquad \text{(VI.2.1)}$$

In equilibrium unidirectional current a pressure difference in phases is equal to the capillary pressure, determined by formulas (VI.1.3) or (VI.1.4):

$$p_2 - p_1 = p_c(s) = \alpha \sqrt{\frac{m}{k}} \cos\Theta J(s). \qquad \text{(VI.2.2)}$$

For a certainty we will index 1 relate to the more wetting phase whose saturation is equal to s. In the problems of the consistent filtration of oil or gas and water the more wetting phase usually is displacing water.

For obtaining the locked system of equations, it is necessary to write the equations of conservation of mass for both phases. Let us isolate in filtration flow certain volume T. The mass of the first phase in it is equal $\int_T m\rho_1 d\tau$ ($d\tau$ - element of volume T). The inflow of the first phase for time dt through surface of S, which limits volume T, composes $-dt\int_S \rho_1\bar{u}_1\bar{n}d\sigma$ ($\bar{n}$ - the vector of normal to the surface S,$d\sigma$ - the cell/element of surface S).

Page 157.

Assuming that in volume T are not contained the sources of mass, we have

$$\frac{d}{dt}\int_T m\rho_1 s\, d\tau + \int_S \rho_1\bar{u}_1\bar{n}\, d\sigma = 0, \qquad \text{(VI.2.3)}$$

whence, converting surface integral into volumetric and taking into

account that volume T is arbitrary, but its boundaries are motionless, we will obtain finally

$$\frac{\partial}{\partial t}(m\rho_1 s) + \operatorname{div}(\rho_1 \bar{u}_1) = 0. \qquad (VI.2.4)$$

Analogously is derive/concluded the equation of conservation of mass for the second phase:

$$\frac{\partial}{\partial t}[m\rho_2(1-s)] + \operatorname{div}(\rho_2 \bar{u}_2) = 0. \qquad (VI.2.5)$$

Thus, we obtain locked system of equations for an $p_i$ and s, since $\rho_1$ and $\rho_2$ they are the functions of pressures $p_1$ and $p_2$, but a change of the porosity in uniform layer depends only on a change in mean pressure $\bar{p} = p_1 s + p_2 (1-s)$ (ethyl alcohol chapter II), but not the pressure of components.

If displaced and displacing phases - slightly-compressible true

liquids, by the effect of incompressibility on the distribution of saturation it is possible to disregard. Actually, characteristic time of the unsteady redistribution of pressure because of compressibility it is of the order of the $t_1 = L^2/\varkappa$, where of the $\varkappa$ - the coefficient of piezoconductivity, L - significant dimension. Characteristic time of displacement - order $t_2 = L/u$, where u is the average rate of filtration. Usually rate of filtration is about $10^{-3}$ cm/s, $L \approx 10^4$-$10^5$ cm., and $\varkappa \approx 10^1$ $cm^2/s$. Therefore $t_1/t_2 \approx 10^{-2}$, from which it is clear that are unsteady the processes of the elastic redistribution of pressure they conclude in the beginning of the course of displacement.

If the liquids and the porous medium can be considered incompressible, we have instead of (VI.2.4) and (IV.2.5)

$$m\frac{\partial s}{\partial t} + \operatorname{div}\bar{u}_1 = 0, \quad m\frac{\partial s}{\partial t} - \operatorname{div}\bar{u}_2 = 0. \qquad \text{(VI.2.6)}$$

The solution of system of equations (VI.2.1), (VI.2.2), (VI.2.6) for two- or three-dimensional cases is very complicated. For the analysis of the common properties of the field of the saturation

with of displacement, let us use the asymptotic approach, based on the smallness of some dimensionless parameters, which enter the conditions of problem (see G. I. Barenblatt's work [26]).

In system of equations and boundary conditions, enter following dimensional determining parameters: $\Delta p$ - the pressure differential between holes or galleries; L is a significant dimension of zone of flow; coefficients $k/\mu_1$ and $k/\mu_2$, and also $p_c^0=\alpha\sqrt{m/k}\cos\Theta$. Let us pass in equations (VI.2.1), (VI.2.2), (VI.2.6) to the dimensionless variables:

$$\frac{p_i}{\Delta p}=P_i;\quad \overline{U}_i=\frac{\overline{u}_i}{u_0};\quad \left(\overline{u}_0=\frac{k\,\Delta p}{\mu_1 L}\right);\quad X=\frac{x}{L};$$

$$Y=\frac{y}{L};\quad Z=\frac{z}{L};\quad \tau=\frac{u_0 t}{L}=\frac{k\,\Delta p}{\mu_1 L^2}t;\quad \varepsilon=\frac{p_c^0}{\Delta p}.$$

Page 158.

This reduces to system of equations

$$\overline{U}_1 = -f_1(s)\,\mathrm{grad}\,P_2 + \varepsilon f_1(s)\,J(s)\,\mathrm{grad}\,s;$$
$$\overline{U}_2 = -\mu_0 f_2(s)\,\mathrm{grad}\,P_2,\quad \mu_0 = \mu_1/\mu_2 \qquad \text{(VI.2.7)}$$

(operator grad it is fulfilled here in alternating/variable X, Y, Z):

$$m\frac{\partial s}{\partial \tau} + \frac{\partial U_{1X}}{\partial X} + \frac{\partial U_{1Y}}{\partial Y} + \frac{\partial U_{1Z}}{\partial Z} = 0;$$
$$m\frac{\partial s}{\partial \tau} - \frac{\partial U_{2X}}{\partial X} - \frac{\partial U_{2Y}}{\partial Y} - \frac{\partial U_{2Z}}{\partial Z} = 0. \qquad \text{(VI.2.8)}$$

. Equations (VI.2.7) are written through $P_2$, since the only displaced

liquid remains continuous in an entire zone of flow, and displace can in the whole regions, where still not reached the front of displacement, to be found in the form of separate drops.

The parameter of $\varepsilon = \frac{p_c^0}{\Delta p}$ is low in the majority of interesting for application/appendices problems, thus it was noted in §1. Therefore the term of $\overline{U}_c = \varepsilon f_1(s) J'(s)\,\mathrm{grad}\, s$ can be essential only in the narrow regions where is great the gradient of saturation. The very fact of the existence of such regions will be established/installed later, thus far let us note that their extent must be small in comparison with the basic exterior precisely because in them is great $|\mathrm{grad}\, s|$, and s - the limited value. This makes it possible to use for the analysis of system (VI.2.7), (VI.2.8) the method of asymptotic union [36a], which entails the use of different scales in the examination of motion in the basic region and in the narrow zones of an abrupt change in the saturation. Similar to this boundary-layer flow of viscous fluid near walls has another three-dimensional/space scale, rather than external flow.

Let us divide the region of filtration to cuter zone, where the $U_c$ - low value, and narrow inner zone or the zones, where as a result of great significance $|\mathrm{grad}\, s|$ the value of $U_c$ cannot be

disregarded.

The smallness of the parameter leads to the natural attempt to investigate current in outer zone, decompose/expanding the unknown solution in power series in this parameter (external resolution). The first term of external expansion we will obtain, by set/assuming $\varepsilon = 0$. The study of the structure of inner zone (internal resolution) will be examined in the following paragraph.

The common properties of external resolution let us examine at first in an example of the one-dimensional problem whose solution is obtained in the locked form.

Page 159.

In the one-dimensional case (plane, radial or spherical) of equation (VI.2.8) taking into account (VI.2.7) with $\varepsilon = 0$ let us write, by returning to dimensional variables, in the form:

$$m\frac{\partial s}{\partial t}-\frac{k}{\mu_1}\frac{1}{x^{\nu-1}}\frac{\partial}{\partial x}\left(x^{\nu-1}f_1(s)\frac{\partial p}{\partial x}\right)=0;$$
$$m\frac{\partial s}{\partial t}+\frac{k}{\mu_2}\frac{1}{x^{\nu-1}}\frac{\partial}{\partial x}\left(x^{\nu-1}f_2(s)\frac{\partial p}{\partial x}\right)=0\qquad(p_1=p_2=p)\qquad\text{(VI.2.9)}$$

($\nu$ = 1, 2, 3 respectively for linear, radial or spherical current).

By deducting the second equation (VI.2.9) from the first and by integrating, we will obtain

$$[f_1(s)+\mu_0 f_2(s)]\frac{\partial p}{\partial x}=\frac{C(t)}{x^{\nu-1}}\qquad\left(\mu_0=\frac{\mu_1}{\mu_2}\right).\qquad\text{(VI.2.10)}$$

. This equality expresses the constancy of the total expenditure/consumption along tube of flow on the strength of the incompressibility of liquids. By determining $\partial p/\partial x$ from

(VI.2.10) and by substituting in any of the equations (VI.2.9), we will obtain one equation for s:

$$\frac{\partial s}{\partial t}+\frac{q(t)}{m}\frac{F'(s)}{x^{\nu-1}}\frac{\partial s}{\partial x}=0, \qquad \text{(VI.2.11)}$$

where $q(t)=-\frac{k}{\mu_1}C(t)$ are the total fluid flow rate through the tube of flow, and

$$F(s)=\frac{f_1(s)}{f_1(s)+\mu_0 f_2(s)}.$$

Function F (s) is equal to the relation of the rate of filtration (or expenditure/consumption) of the displacing phase to the total rate of filtration (or to the total expenditure/consumption). Function F (s) is conventionally designated as the function of phase distribution.

Let us introduce the new independent variables: $Q=\int\limits_{t_0}^{t}\frac{q(t)}{m}dt,\ W=\frac{x^{\nu}}{\nu}.$

Value W can be considered as volume of tube of flow between sections $x_0$ and x (specifically, with $\nu$ = 1 and q (t) = const alternating/variable Q is proportional to time, but W = x). Then instead of (VI.2.11) we have

$$\frac{\partial s}{\partial Q}+F'(s)\frac{\partial s}{\partial W}=0. \qquad \text{(VI.2.12)}$$

To this equation in the partial derivatives of the first order corresponds the following system of characteristic equations:

$$\frac{dQ}{1}=\frac{dW}{F(s)}=\frac{ds}{0}. \qquad \text{(VI.2.13)}$$

The general solution of system (VI.2.13) is represented in the form:

$$s = C_1;$$
$$W = QF'(s) + C_2. \quad (\text{VI.2.14})$$

Page 160.

Thus, along the characteristics of equation (VI.2.12) s = const and in plane (W, Q) characteristic they are straight lines. Physically this means that each value of saturation s is spread at the "rate" dW/dQ, proportional F' (s). In the case of motion by plane waves $\nu$ = 1, and dW/dQ there is true velocity of propagation of this value of saturation.

On the basis of equalities (VI.2.14) the general solution of equation (VI.2.12) can be formally written in the form:

$$W = QF'(s) + W_0(s), \qquad \text{(VI.2.15)}$$

where function $W_0$ (s) it corresponds to the initial distribution of saturation (with $Q = 0$, i.e., $t = t_0$).

On holes (or the galleries) through which is forced the displacing liquid, must be assigned the conditions, which define the classification of the forced liquid. If is forced one (displacing) liquid, then this condition is record/written in the form:

$$u_2 = -\frac{k}{\mu_2} f_2(s) \frac{\partial p}{\partial x} = 0. \qquad \text{(VI.2.16)}$$

Since $\partial p/\partial x \neq 0$, since is not equal to zero the total expenditure/consumption, of (VI.2.5) follows $f_2(s) = 0$, $s \leqslant s_{**}$. Let at the moment everywhere in layer the saturation of the displacing phase

be lower than $s_{**}$ ($s_{**}$ - the saturation with which the displaced phase becomes motionless). This condition is satisfied almost in all problems, which make physical sense. Then with t > 0 on boundary will be fulfilled condition

$$s = s_{**}. \qquad \text{(VI.2.17)}$$

Actually, if one assumes that at certain point of the boundary of $s > s_{**}$, that on the strength of the continuity of saturation near this point of boundary must exist the whole region of the finite dimensions, in which $s > s_{**}$ and $u_2 = 0$. On the strength of the equation of continuity for the displaced phase (VI.2.6) hence follows $\partial s/\partial t = 0$. This contradicts the condition of $s \leqslant s_{**}$ with t = 0. But if is assigned the relation of the consumption of two phases on the surface of forcing λ, then boundary conditions takes the form:

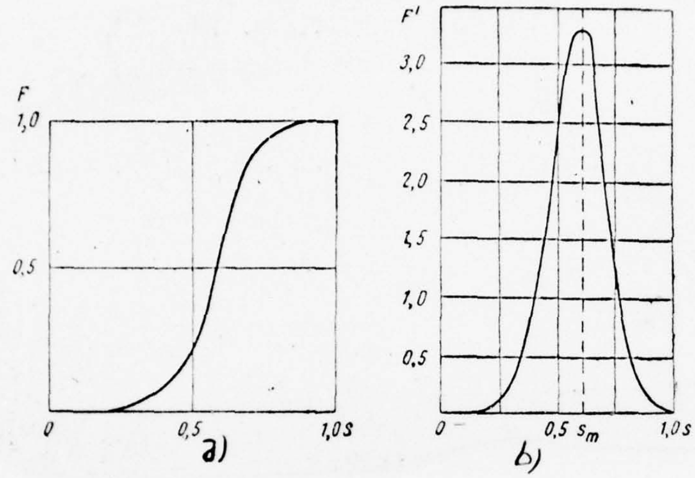
$$\mu_0 \frac{f_2(s)}{f_1(s)} = \lambda, \qquad \text{(VI.2.18)}$$

whence also is determined the saturation on boundary, since the relative permeability $f_1$ (s) and $f_2$ (s) are known.

Let us return to the analysis of the general view of solution (VI.2.15).

Figure VI.7a, b depicts the typical curves F (s) and F' (s). Function F' (s) has a maximum at certain point of $s_m$. Therefore in accordance with formula (VI.2.15) two different values of saturation can have identical velocity of propagation. In connection with this the obtained according to formula (VI.2.15) dependence of saturation on W can stop are not identical, great significance, "pass" less, as this is shown in Fig. VI.8.

Fig. VI.7.



The ambiguity of the formal solution, obtained from (VI.2.15), means that the continuous solutions of the problem of displacement in the assigned initial condition do not exist. In order to obtain the solution, which makes physical sense, it is necessary to insert the discontinuity/interruptions (jumps) of saturation, i.e., the surface on which the value of saturation is changed by jump. Such jumps can exist, also, under the initial conditions.

With an increase of Q, the initial smooth distribution of saturation is transformed as this evidently in Fig. VI.8. At certain torque/moment (i.e. with certain Q) tangent to the curve of s (W) becomes vertical. From this time on, appears and is spread the jump of saturation. Position of the jumps of the saturation previously is unknown. On jumps (discontinuity surfaces) must be fulfilled the continuity conditions of pressure and conservation of mass of each of the driving phases.

Let us derive conditions on jumps for the general case of nonuniform two-phase current. The first of these conditions is record/written in the form:

$$p^{(1)} = p^{(2)}. \qquad (VI.2.19)$$

Let us examine now the conditions of the conservation of mass each of the phases during passage discontinuity surface (jump) through certain element of volume of the porous medium (Fig. VI.9), cut out along the normal to discontinuity surface.

Page 162.

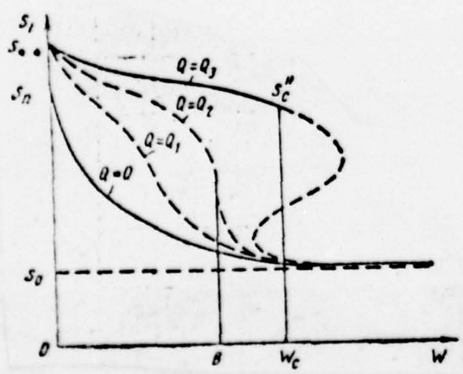



Fig. VI.8.

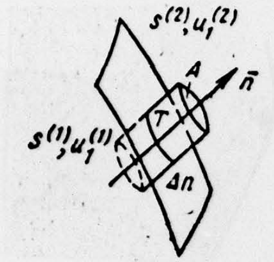



Fig. 9.

On the strength of the continuity of pressure, the compressibility of liquids for conditions on jump is unessential. Let us use to this cell/element the equation of conservation of mass (VI.2.3). We have

$$\frac{d}{dt}\left(\int\limits_{T} ms\,d\tau\right)=mV_n(s^{(1)}-s^{(2)})\,A+o\,(T),$$

where A - the cross-sectional area of cell/element (index 1 from above designated values after jump, index 2 - before the jump); $V_n$ - - the speed of the displacement/movement of jump along the normal to it; about (T) is the value, which vanishes is faster than T.

Fluid flow through sections, parallel discontinuity surfaces, is equal $(u_{1n}^{(1)}-u_{1n}^{(2)})\,A\,(u_{1n}$ - the projection of the rate of filtration of the first phase on normal to the surface of jump). The flow, connected with peripheral component of velocity, is vanishingly small in comparison with $u_{1n}A$ with tendency Δn to zero on the strength of condition (VI.2.19).

The condition of the conservation of mass of the first liquid

will take then the form:

$$mV_n(s^{(1)}-s^{(2)})=u_{1n}^{(1)}-u_{1n}^{(2)}. \quad \text{(VI.2.20)}$$

Equality (VI.2.20) can be rewritten in the form:

$$V_n=\frac{u_{1n}^{(1)}-u_{1n}^{(2)}}{m\left(s^{(1)}-s^{(2)}\right)}. \quad \text{(VI.2.21)}$$

The condition of the conservation of mass of the second liquid also is reduced to expression (VI.2.20), since

$$u_{1n}^{(1)}+u_{2n}^{(1)}=u_{1n}^{(2)}+u_{2n}^{(2)}. \quad \text{(VI.2.22)}$$

Let us return to the investigated one-dimensional problem. From determination of function q (t) and F (s) it is possible to

write expressions for the rates of filtration of the displacing phase behind, also, in front of the jump:

$$u_1^{(i)} = \frac{q(t)}{x^{\nu}} F(s^i) \qquad (i = 1, 2), \qquad \text{(VI.2.23)}$$

where index 1 it is related to values after jump, index 2 is before the jump.

Page 163.

then we will obtain the following expression for the rate of the jump of $V_n = V = dx/dt$:

$$\frac{dx}{dt} = \frac{q(t)}{mx^{\nu-1}} \frac{F(s^{(1)}) - F(s^{(2)})}{s^{(1)} - s^{(2)}}. \qquad \text{(VI.2.24)}$$

By transfer/converting to alternating/variable W and Q, we will obtain

$$\frac{dW}{dQ}=\frac{F(s^{(1)})-F(s^{(2)})}{s^{(1)}-s^{(2)}}. \qquad \text{(VI.2.25)}$$

The distribution of saturation on both sides of jump is described by formula (VI.2.15). The position of the jump of $W_c$ and saturation on the jump of $s_c=s^{(1)}$ can be determined by calculation by consecutive spaces, on the strength of the initial position of jump (i.e. the place of appearance by vertical tangential), on formula (VI.2.25).

Let us derive the differential equation, which describes a change in the saturation on jump depending on Q. For a saturation with jump as for any value of saturation, is fulfilled the relationship/ratio (VI.2.15):

$$W_c=QF'(s_c)+W_0(s_c). \qquad \text{(VI.2.26)}$$

It is obvious, accordingly (VI.2.26), $s_c = s^{(1)}$ is changed with change Q (i.e. in the course of time). Differentiating (VI.2.26) with respect to Q, we have

$$\frac{dW_c}{dQ} = F'(s_c) + [F''(s_c)Q + W_0'(s_c)]\frac{ds_c}{dQ}. \qquad \text{(VI.2.27)}$$

By equating expressions for the velocity of propagation of the jump of saturation (VI.2.25) and (VI.2.27), let us arrive at the following equation for an $s_c$:

$$\frac{ds_c}{dQ} = \frac{F(s_c) - F(s^{(2)}) - F'(s_c)(s_c - s^{(2)})}{(s_c - s^{(2)})[F''(s_c)Q + W_c'(s_c)]}. \qquad \text{(VI.2.28)}$$

The value of $s^{(2)}$ in equation (Vi.2.28) is function Q $s_c$. It is determined from the condition of $W(s^{(2)}) = W(s_c) = W_c$ in accordance with formula (VI.2.15):

$$QF'(s_c) + W_0(s_c) = QF'(s^{(2)}) + W_0(s^{(2)}). \qquad (VI.2.29)$$

For the solution to equation (VI.2.28) the initial values of $s_c$ and Q are determined at that point where dW/ds, correspond to formula (VI.2.15), becomes zero for the first time.

From equation (VI.2.28) it follows that on the jump of saturation, on both sides of which the $s_c$ and $s_0 = s^{(2)}$ are constant (stationary jump), must be fulfilled condition

$$\frac{F(s_c) - F(s_0)}{s_c - s_0} = F'(s_c). \qquad \text{(VI.2.30)}$$

Page 164.

This condition (obtained for the first time in the work of Bakley and Leverett [134] means that the velocity of propagation of jump is equal to the velocity of propagation of saturation on the jump of $s_c$.

Let us examine separately the case when the initial saturation s (W, 0) = $s_0$ is constant in all layer. Then $W_0$ (s) = 0 with s > $s_0$ and $W_0$ (s) is not defined with s < $s_0$. From formula (VI.2.15) we have in this case

$$W = QF'(s) \qquad (s > s_0).$$

Let us assume that $s_0 < s_m$ (see Fig. VI.7b), and jump appears. Under given conditions in equation (VI.2.28) the variables are divided.

Let us designate $s_c - s_0 = y,\ F(s_c) - F(s_0) = \psi(y)$. Then from equation (VI.2.28), integrating, we obtain

$$\psi(y) - y\psi'(y) = \frac{C}{Q}; \quad C = \text{const.} \qquad \text{(VI.2.31)}$$

Since with $Q \to 0$ (at the moment), the left side of the equality (VI.2.31) remains that which was limited, C must become zero. Consequently, the jump is stationary. Returning to the variables of $s_c$ and $s_0$ we obtain, that on jump with any Q is

fulfilled the relationship/ratio (VI.2.30).

Let the initial distribution of saturation be such, that $W'_0$ (s) < 0 with all s, and with $W \to \infty$ s (W, 0) $\to s_0$. Then the solution examined above for case $s_0$ = const will be fulfilled asymptotically in $Q \to \infty$, since the $W'_0(s_c)$ is limited.

The examined solution and condition (VI.2.30) was obtained Bakley and Leverett [134]. The study of the motion of the jumps of saturation is carried out by S. N. Buzinov and I. A. Charn [36].

In the general case of non-one-dimensional problem, the systems (VI.2.7) and (VI.2.8) even with $\varepsilon$ = 0 no longer are reduced to one equation for a saturation. It is necessary to determine p and s together. In this case, the boundary and initial conditions for p the same as in the problems of the filtration of homogeneous liquid.

Conditions for s take the form (VI.2.17) or (VI.2.18). Furthermore, on the jumps whose position is previously unknown, must be fulfilled conditions (VI.2.19), (VI.2.22) and (VI.2.23). The

solutions to non-one-dimensional problems can be obtained only numerically on EVM [ - computer].

Page 165.

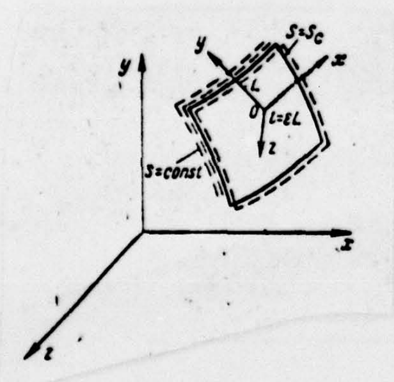



Fig. VI.10.

§3. Structure of the jump of saturation. Equation is the Rapoport-fox, stabilized zone. In an example of the solution of Bakley-Leverett, it is evident that if we in the problem of the displacement of oil by water are restricted to the first term of external expansion, i.e., not to consider a capillary pressure difference, then it is necessary to introduce to the surfaces on which the saturation suffers discontinuity/interruption.

In order to describe the distribution of saturation in narrow regions, near jump, it is necessary to obtain the "internal resolution" of the solution of the problem of displacement on the basis of complete system of equations (VI.2.7), (VI.2.8). For the construction of the first term of internal expansion, let us introduce in the vicinity of certain point of discontinuity surface (i.e. the jump of saturation) the local instantaneous Cartesian coordinate system with center at the point O discontinuity surface. $x$-axis is directed along the normal to discontinuity surface and let us introduce along this axis scale $l = \epsilon L$, i.e., let us place $X = x/l$, by retaining along other axes scale L (Fig. VI.10). Time scale let us take as $t_1 = l/u_0$ and is placed $\tau = t/t_1$. In other respects let us preserve the same dimensionless parameters and the variables, as in equations (VI.2.7) and (VI.2.8).

Let us write system of equations in a dimensionless form. Generalized law of the darcys:

$$
\begin{aligned}
&U_{1X} = -\frac{1}{\varepsilon} f_1(s) \frac{\partial P_1}{\partial X} \quad && U_{2X} = -\frac{\mu_0}{\varepsilon} f_2(s) \frac{\partial P_2}{\partial X};\\
&U_{1Y} = -f_1(s) \frac{\partial P_1}{\partial Y} \quad && U_{2Y} = -\mu_0 f_2(s) \frac{\partial P_2}{\partial Y};\\
&U_{1Z} = -f_1(s) \frac{\partial P_1}{\partial Z} \quad && U_{2Z} = -\mu_0 f_2(s) \frac{\partial P_2}{\partial Z};\\
&P_2 - P_1 = \varepsilon J(s) \quad && \mu_0 = \frac{\mu_1}{\mu_2}.
\end{aligned}
\tag{VI.3.1}
$$

Equations of the continuity:

$$m\frac{\partial s}{\partial \tau}+\frac{\partial U_{1X}}{\partial X}+\varepsilon\frac{\partial U_{1Y}}{\partial Y}+\varepsilon\frac{\partial U_{1Z}}{\partial Z}=0;$$
$$m\frac{\partial s}{\partial \tau}-\frac{\partial U_{2X}}{\partial X}-\varepsilon\frac{\partial U_{2Y}}{\partial Y}-\varepsilon\frac{\partial U_{2Z}}{\partial Z}=0. \qquad \text{(VI.3.2)}$$

By substituting (VI.3.1) in (VI.3.2) and by reject/throwing the terms of order $\varepsilon$ and $\varepsilon^2$, we will obtain

$$m\frac{\partial s}{\partial \tau}-\frac{1}{\varepsilon}\frac{\partial}{\partial X}\left(f_1(s)\frac{\partial P_1}{\partial X}\right)=0; \qquad \text{(VI.3.3)}$$
$$m\frac{\partial s}{\partial \tau}+\frac{\mu_0}{\varepsilon}\frac{\partial}{\partial X}\left(f_2(s)\frac{\partial P_1}{\partial X}\right)+\mu_0\frac{\partial}{\partial X}\left(f_2(s)J'(s)\frac{ds}{\partial X}\right)=0.$$

Page 166.

Let us exclude $\partial P_1/\partial X$ from equations (VI.3.3), which gives finally

$$m\frac{\partial s}{\partial \tau}-w\frac{\partial F(s)}{\partial X}+\mu_0\frac{\partial}{\partial X}\left(f_2(s)\,F(s)\,J'(s)\frac{\partial s}{\partial X}\right)=0, \qquad \text{(VI.3.4)}$$

where

$$w(\tau)=\frac{u_1+u_2}{u_0}=-\frac{1}{\varepsilon}\left[f_1(s)+\mu_0 f_2(s)\right]\frac{\partial P_1}{\partial X}+\mu_0 f_2 J'(s)\frac{\partial s}{\partial X}=$$
$$=U_{1X}+U_{2X}. \qquad \text{(VI.3.5)}$$

. (V.3.1) - (VI.3.2) came and to one-dimensional equation (VI.3.4), because the radius of curvature of discontinuity surface is of the order L, and in the taken scale $l$ this surface as and surface

$s = \text{const}$, it is replaced by planes. Equality (VI.3.5) means that within the limits of the zone of the jump, where the motion can be considered one-dimensional, the total rate of filtration of both phases along X-axis, w is the value, which depends only on time as during motion in cylindrical tube of flow.

Value w in equation (VI.3.4) is located from external resolution as the dimensionless total rate of filtration through the discontinuity surface in point 0. The equation (VI.3.4), which describes the one-dimensional displacement of the nonmiscible liquids, is called the equation of rapoport - Lis [152]. In3 since the time scale in the internal expansion $t_1$ considerably less than in external, and rate w is determined by external resolution, during the study of internal resolution it is possible to count $w(\tau) =$ = const. In view of a difference in the time scales in external and internal resolution, is sufficient to use the steady-state solution of the Cauchy problem for equation (VI.3.4), i.e., to rely

$$s = s(\bar{x}), \quad \bar{x} = X - V^0\tau \qquad \left(V^0 = \frac{V}{u_0}\right). \qquad \text{(VI.3.6)}$$

In other words, to scale of time of internal resolution the process of displacement continued very for long, and it it is possible to consider establish/installed in the coordinate system, connected with jump. In this case, on the strength of a difference in the scales $l$ and L, must be fulfilled boundary conditions

$$s(-\infty) = s^{(1)} = s_c; \qquad s(+\infty) = s^{(2)} = s_0, \qquad \text{(VI.3.7)}$$

where the $s^{(1)} = s_c$ and $s^{(2)} = s_0$ ) - saturation after and before the jump, determined from external resolution [in the problem of Bakley-Leverett they are connected by relationship/ratio (VI.2.30) ]. Parameter $V^0$ in equality (VI.3.6) is, obviously, the velocity of propagation of discontinuity surface, determined by formulas (VI.2.21) or (VI.2.24).

Page 167.

By utilizing (VI.3.6), we will obtain instead of (VI.3.4) equation

$$-mV^0 \frac{ds}{d\bar{x}} - w \frac{dF(s)}{d\bar{x}} + \mu_0 \frac{d}{d\bar{x}} \left[ f_2(s) F(s) J'(s) \frac{ds}{d\bar{x}} \right] = 0. \qquad \text{(VI.3.8)}$$

Integration gives

$$-mV^0 s - wF(s) + \mu_0 f_2(s) F_s J'(s) \frac{ds}{d\bar{x}} = c = \text{const.} \qquad \text{(VI.3.9)}$$

From the condition of the $s=s_c$ with of $\bar{x}\to-\infty$, taking into account that in this case $ds/d\bar{x}=0$, we have

$$c = -mV^0 s_c - wF(s_c). \qquad \text{(VI.3.10)}$$

Let us note that, since the value cf V is determined by formula (VI.2.24), the second condition (VI.3.7) will be satisfied

automatically. Substituting the value of c from (VI.3.10) in (VI.3.9) and solving relative to $d\bar{x}/ds$, we will obtain

$$\frac{d\bar{x}}{ds}=\frac{w\mu_0 f_2(s)F(s)J'(s)}{V^0(s-s_0)-[F(s)-F(s_0)]w}. \qquad (VI.3.11)$$

If we integrate equation (VI.3.11) for s, accepting reference point then so that with $\bar{x}=\bar{x}_1$ would be the $s=s_1$ where $s_0<s_1<s_c$, we will obtain, utilizing for $V^0$ formulas (VI.2.24) and (VI.3.5) with $s^{(1)}=s_c$, $s^{(2)}=s_0$:

$$\left(V=\frac{u_0 w}{m}\frac{F(s_c)-F(s_0)}{s_c-s_0};\quad V^0=\frac{w}{m}\frac{F(s_c)-F(s_0)}{s_c-s_0}\right);$$

$$\bar{x}-\bar{x}_1=\int\limits_{s_1}^{s}\frac{f_2(s)F(s)J'(s)\,ds}{[F(s_c)-F(s_0)]\frac{s-s_0}{s_c-s_0}-F(s)+F(s_0)}. \qquad (VI.3.12)$$

If correctly the assumption about the steadiness of the jump of saturation and V is determined from formula (VI.2.24), we will obtain another notation for $\bar{x}(s)$:

$$\bar{x} - \bar{x}_1 = \int_{s_1}^{s} \frac{f_2(s)\, F(s)\, J'(s)\, ds}{F'(s_c)(s - s_0) - F(s) + F(s_0)}. \qquad \text{(VI.3.13)}$$

Integrals (VI.3.12) and (VI.3.13) describe the transition zone of infinite extent, what is the consequence of the taken approximation. Actually for determining the width of zone, it is necessary to undertake according to formulas (VI.3.12) and (VI.3.13) the distance between points with saturation $s_0 + \delta$ and $s_0 - \delta$, where $\delta$ is a low, but finite quantity. Then the dimensionless width of transition zone there will be the order of several ones, and dimensional width - order $\ell$, i.e., $\frac{p_c}{\Delta p} L$ or $\frac{\alpha \sqrt{k}}{\mu_1 \mu_0}$. The typical distribution curve of saturation in transition zone is given in Fig. VI.11.

We examine the distribution of saturation in transition zone depending on the form of the function F (s) and J' (s) and the values of $s_c$ and $s_0$.

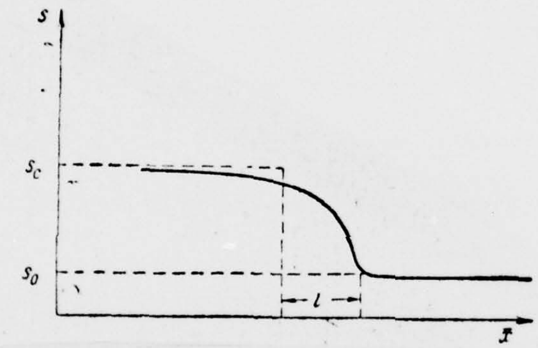



Fig. VI.11.

With $s$, close to $s_c$, the denominator of integrand is of the order

$$(s - s_c)\left\{\frac{F(s_c) - F(s_0)}{s_c - s_0} - F'(s_c)\right\} + a_1 (s - s_c)^2,$$

a numerator is finite quantity. Therefore, if the $\frac{F(s_c) - F(s_0)}{s_c - s_0} \neq F'(s_c)$, then for large negative $\bar{x}$ we have an $\bar{x} \approx B \ln (s - s_c)$. But if $\frac{F(s_c) - F(s_0)}{s_c - s_0} = F'(s_c)$ (case of Bakley-Leverett), then $\bar{x} \approx C (s_c - s)^{-1}$ ($a_1$, B, C - constants).

If $s_0$ and F ($s_0$) do not become zero and dJ/ds with $s = s_0$ of course, then from (VI.3.12) for $s$, close to $s_0$, follows $\bar{x} - \bar{x}_1 \approx A \ln (s - s_0)$.

Let now F ($s_0$) = 0 and, therefore, of $s_* \geqslant s_0 \geqslant 0$. Then instead of (VI.3.12) we obtain

$$\bar{x} - \bar{x}_1 = \int_{s_1}^{s} \left[\frac{F(s_c)}{s_c - s_0} - \frac{F(s)}{s - s_0}\right] \frac{f_2(s) F(s) J'(s)}{s - s_0} ds. \quad \text{(VI.3.14)}$$

Function $f_1$ (s) with s, close to $s_*$, has the form of the $b(s - s_*)^\beta$, where of the $\beta > 2$, $F(s) \approx M(s - s_*)^\beta$, b and M - constants. Then with s, close to $s_*$, if $s_* \neq s_0$, that

$$\bar{x} - \bar{x}_* = N \int_{s_*}^{s} (s - s_*)^\beta J'(s)\, ds. \qquad \text{(VI.3.15)}$$

With $s \to s_*$ value J' (s) increases more slowly than $(s - s_*)^{-(\beta+1)}$, integral (VI.3.15) converges, and the value of $s = s_*$ is reached at the finite value of $\bar{x}$ (Fig. VI.12), and from $s_*$ to $s_0$, appears the jump of saturation. On this jump as on any jump of saturation, must be fulfilled the condition (VI.2.12).

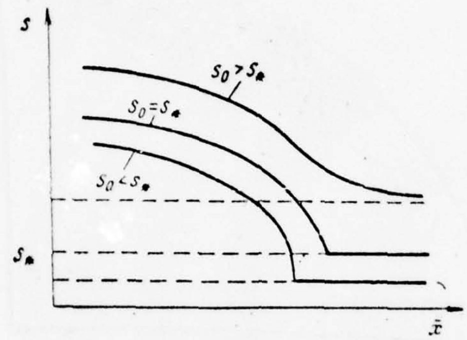



Fig. VI.12.

Is checked its execution. In our case of $V_n = V,\ s^{(1)} = s_*,\ s^{(2)} = s_0,\ u_{1n}^{(2)} = 0.$ Then condition (VI.2.12) will take the form:

$$u_1^{(1)} = u_1(s - s_*) = V_m(s_* - s_0). \qquad \text{(VI.3.16)}$$

By comparing formulas (VI.3.3) and (VI.3.4), it is possible to obtain

$$u_1 = u_0\left[wF(s) - \mu_0 f_2(s) F(s) J'(s) \frac{ds}{d\bar{x}}\right]. \qquad \text{(VI.3.17)}$$

Now it is not difficult to ascertain that taking into account (VI.3.17) from the equation (VI.3.11) with of $s = s_*$ follows the condition (VI.3.16).

If $s_0 = s_*$, that instead of the formula (VI.3.15) we have with s, close to $s_*$:

$$\bar{x} - \bar{x}_* = N_1 \int_{s_*}^{s} (s - s_*)^{\beta - 1} J'(s)\, ds. \qquad \text{(VI.3.18)}$$

Value s turns into $s_*$ at the finite value of $\bar{x}$ in such a case, when integral (VI.3.18) converges.

The existence of the solutions of form (VI.3.12) and (VI.3.13) shows that during the constant velocity of displacement the distribution of saturation in transition zone is stationary. It is experimental this stationary transition zone during the displacement of oil by water in the ducts, filled by sand, it was reveal/detected by Tervilliger, etc. [158] and is subsequently minutely investigated in the works of Rapoport and Lis [152] and D. A. Efros and V. P. Onopriyenkya [128]. In connection with the steadiness of the distribution of saturation, this zone was called the name the stabilized zone. Filtration in the stabilized zone was investigated in the works Jones-of Parr and Kolkhaun [141] ~~[144]~~, and also of V. M. Ryzhik, I. A. Charnyy, Chen'-Chzhun-Syan [99a].

The distribution of saturation in the stabilized zone is establish/installed as a result of the joint action of the forces of liquid resistance, gravitation and capillary forces. All these forces are located in equilibrium at the constant velocity of displacement. Here there is an analogy with the hydrostatic equilibrium of liquid in the porous medium when the phase boundary is that which was washed away under the action of capillary forces due to the difference in the size/dimensions of pore channels. Roughly estimating force intensity, which act on liquid in transition zone, it is possible to say that the capillary pressure, which causes diffusion of front, is of the order $\alpha$ $(1/r_0-1/r_1)$, where $r_0$ and $r_1$ are minimum and maximum "radii of the pores" of the medium.

Page 170.

The jump/drop pressures, necessary for overcoming of liquid resistance, and gravitational force in zone by length $l$ are proportional to $l$. Therefore at constant velocity the size/dimension of the zone in which the "srabatyvaetsr" capillary jump/drop, remains constant.

The possibility of the description of the process of displacement on a large scale with the aid of external resolution (for example with the use of solution of Bakley-Leverett) is connected only with the smallness of the parameter of $\varepsilon = \frac{p_c^0}{\Delta p}$. If the boundary conditions of exterior problem are such, that the parameter is low, then it remains small even in such a case, when capillary pressure depends, besides saturation, and on other parameters, since the capillary pressure connected mainly with phase distribution in pores and during any distribution remains that which was limited. On the contrary, during the analysis of the structure of transition zone in the present paragraph it was considered that the capillary pressure and the relative permeability are the functions only of saturation.

G. I. Barenblatt [26] noted that in internal transition zone the use of an assumption about phase permeability and capillary pressure as the universal functions of instantaneous saturation is unequal due to the nonequilibrium processes of the redistribution of phases in pores. Instead of this it is assumed that at each torque/moment with each saturation s there are volumes displacing and displaced phases,

equal $\tau_0 \frac{\partial s}{\partial t}$, participates in motion (here $\tau_0$ is certain parameter, which has the dimensionality of time, depending on the properties of the porous medium). These volumes are taken by the processes of redistribution, in connection with which of the pore where they are located, as "are closed". Then in the first approximation, it is possible to assume that the $f_1 = f_1^0\left(s - \tau_0 \frac{\partial s}{\partial t}\right)$, $f_2 = f_2^0\left(s + \tau_0 \frac{\partial s}{\partial t}\right)$, where of the $f_1^0(s)$ and $f_2^0(s)$ are usual "equilibrium" relative permeability. In work [26] were obtained the expressions for the distribution of saturation in the stabilized zone during the made assumptions on the "delay" of relative permeability.

As show experiments, the stabilized zone at the constant velocity of displacement always is formed through sufficiently long time. The measurements of the distribution of saturation in the stabilized zone it can aid to explain, within which limits are valid the assumptions, made during the derivation of formulas (VI.3.12) and (VI.3.13), i.e., assumption about the unique dependence of the functions of $f_i(s)$ and J (s) of saturation and about their independence of rate of filtration.

Page 171.

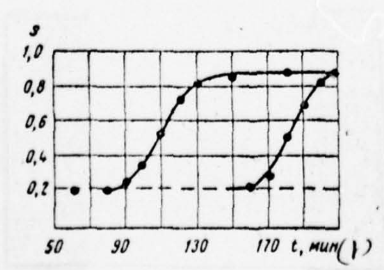
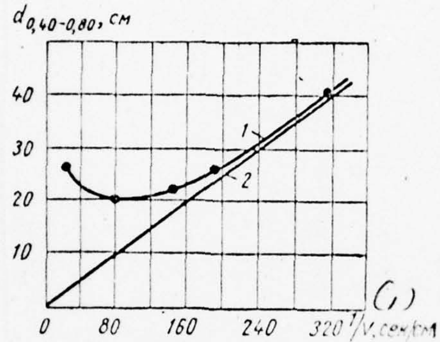



Fig. VI.13.

Key: (1). min.

Fig. VI.14.




Key: (1). s/sm.

Such experiments were carried out by V. M. Ryzhik together with V. N. Martos. From horizontal ducts 170 cm. long, filled by quartz sand with permeability 10 d and porosity 0.40, was displaced the air by water at atmospheric pressure at constant velocity. The distribution of saturation was measured by electrical resistance. The velocity of displacement $u_0$ was changed within limits from 0.0011 to 0.020 cm/s. The initial saturation $s_0$ was equal to 0.21.

The experiments showed that during displacement at constant rate of change of the saturation in different points along the length of model virtually is repeated with shift/shear in time, proportional to the rate of displacement, i.e., is formed the stabilized zone. The typical curves of s (t) at the points, distant from each other on 58 cm., at the rate of front 0.013 cm/s are shown in Fig. VI. 13.

If the relative permeability and the capillary pressure are the functions only of saturation, then in accordance with formulas (VI.3.12) and (VI.3.13) the dimensionless length of the stabilized zone $\delta$ does not depend on rate. This means that dimensional length of this zone, equal $d = \delta \ell$, inversely proportional to rate, i.e.,

$$d = \delta l = \delta \frac{p_c^0}{\Delta p} L = \frac{\delta\alpha \sqrt{km} \cos\Theta}{u_0\mu}. \qquad \text{(VI.3.19)}$$

Figure VI.14 shows the dependence of the length of the stabilized zone d on 1/V (V - the speep of the front of displacement), obtained in the experiments described above (is curve 1); the length of the stabilized zone was defined as distance between points with saturation 0.40 and 0.80. From curve/graph evident (is curve 2) that with large 1/V (low speeds) d - is approximately proportional 1/V as it follows from formulas (VI.3.12) and (VI.3.13). However, at value 1/V about 100 s/cm, is minimum d, and further again is observed an increase in the stabilized zone. Apparently, this increase d is connected with the nonequilibrium of flow and the process lag of the redistribution of phases in pores. By the diagram of the "delaying" relative permeability, as shown in work [26], is obtained precisely this dependence of d on the speed when the value of $\tau_0 \frac{\partial s}{\partial t}$ becomes comparable with s.

Page 172.

From the formulas (VI.3.12) and (VI.3.13) it is possible to obtain also expressions for an which allow, knowing from experiment the distribution of saturation, to find the "dynamic" dependence of capillary pressure on saturation (in this case it is necessary to assume that the relative permeability barely depend on speed. This assumption is correct, if the displacing phase is water, and displaced - air whose ductility/toughness/viscosity is negligible). It turned out that at low speeds the dynamic curves coincide with static curve, and with large - they lie/rest the lower its, the greater speed.

4. Capillary impregnation and the self-similar problems of the displacement of immiscible liquids.

In the preceding/previous paragraph was examined the action of capillary forces near the front of the displacement of the nonmiscible liquids from the porous medium. Capillary forces become essential, also, in other cases when in the porous medium on the strength of its inherent heterogeneity or under the effect of the

heterogeneity of flow are created the considerable local gradients of saturation. In this case, under the action of capillary forces, occurs the redistribution of phases, since the gradient of capillary pressure can be close to the gradient of external pressure, and in a number of cases considerably exceed this gradient. The processes of redistribution (role of which in the displacement of the nonmiscible liquids will be in more detail examined in chapters VII and X) is most simple traced in an example of capillary impregnation, i.e., the filtration, which proceeds under the action only of capillary forces.

It is possible to isolate two idealized process, in which the capillary forces are only motive power [99.100].

1. Let the cylindrical specimen/sample of the porous medium have impenetrable lateral surface. Initially specimen/sample is filled by gas (having negligible ductility/toughness/viscosity). At moment one of the end/leads of the specimen/sample is led to contact with the wetting liquid which begins to be absorbed into specimen/sample. Further we will assume that pressure in gas (initial pressure in specimen/sample) and fluid pressure outside the porous medium are identical.

As into 3, we consider that liquid is the continuous displacing phase and the filtration occurs in one direction. According to the already mentioned considerations the liquid, which moves in the porous medium under the action of capillary forces, can be considered as incompressible. The filtration of continuous phase is described by the generalized law of darcys in the form:

$$u_1 = -\frac{k}{\mu_1} f_1(s) \frac{\partial p_1}{\partial x}. \qquad \text{(VI.4.1)}$$

Page 173.

Since the viscosity of gas is small in comparison with the viscosity of liquid, for gas phase it is possible to place $p_2 = p_0 =$ const. For the unidimensional movement of continuous liquid phase in question is fulfilled the relationship/ratio

or

$$p_2 - p_1 = p_c(s) \qquad \text{(VI.4.2)}$$

$$p_1 = p_0 - p_c(s) \qquad \text{(VI.4.3)}$$

(we leave thus far aside nonequilibrium effects).

Substituting this expression for $p_1$ in equality (VI.4.1), we obtain

$$u_1 = \frac{k}{\mu_1} f_1(s)\, p_c'(s) \frac{\partial s}{\partial x} = -a^2 m \frac{\partial H}{\partial x}, \qquad \text{(VI.4.4)}$$

where

$$a^2 = \frac{\alpha}{\mu_1} \sqrt{\frac{k}{m}}; \quad H(s) = -\int_0^s f_1(s)\, J'(s)\, ds;$$

J (s) - the function of Leverett [see formula (VI.1.4)].

The equation of continuity for a liquid in the one-dimensional case in question takes the usual form:

$$m \frac{\partial s}{\partial t} + \frac{\partial u_1}{\partial x} = 0. \qquad \text{(VI.4.5)}$$

By substituting here expression (VI.4.4), we will obtain the following equation for s:

$$\frac{\partial s}{\partial t} - a^2 \frac{\partial H}{\partial x^2} = 0. \qquad \text{(VI.4.6)}$$

In the curves of J (s), obtained by means of impregnation (for example by the absorption of liquid into the vertical column of the porous medium), always there is this value $s = s^* \leqslant 1$, so $J(s^*) = 0$. On the strength of the continuity of fluid pressure upon transition through the boundary of the porous medium and since the pressure in free liquid is equal $p_0$, then on the boundary of the porous medium must be made condition $p_1 = p_0$, whence $p_c = 0$ and J (s) = 0. consequently, in entrance (where let us accept x = 00 it will be s = s * (if we disregard compressibility).

In exit section, obviously, the displacing liquid is motionless, since the escape of liquid from pore channel cannot occur under the action of some capillary forces alone (for the discharge of liquid must occur the rotation of meniscus at outcrop, which will lead to a sign change of capillary pressure and the cessation of motion). In accordance with formula (VI.4.4) the equality to zero of rate of filtration means that in exit section ($x = l$)

$$f_1(s) = 0 \text{ или } \frac{\partial s}{\partial x} = 0, \qquad \text{(VI.4.7)}$$

since $dp_c/ds$ into zero does not turn.

Page 174.

The first of the conditions (VI.4.7) is made to the approach of liquid to the exit section when the $s \leqslant s_*$ (where the $s_*$ are "immobile" saturation), and the second after approach.

Let us examine the case when $l \to \infty$. Then the only dimensional determining parameter for the distribution of saturation proves to be $a^2$. The dimensionality of this parameter is $L^2/T$. If, furthermore, the initial saturation $s_0$ = const, the problem becomes self-similar and s is the function of alternating/variable $\xi = x/a\sqrt{t}$. Equation (VI.4.6) is transformed into the ordinary differential equation of form

$$\frac{\xi}{2}\frac{ds}{d\xi} + \frac{d^2H}{d\xi^2} = 0. \qquad \text{(VI.4.8)}$$

The dependences of relative permeability for the wetting phase $f_1$ (s) and the function of Leverett J (s) can be approximated by formulas

$$f_1(s) = b(s - s_*)^{\beta}, \quad (f_1(s) \equiv 0 \ \overset{(1)}{\text{при}} \ s < s_*),$$
$$J(s) = C_1 - B_1(s - s_*)^{\alpha_1} \ \overset{(2)}{\text{или}} \quad J(s) = B_2(s - s_*)^{\alpha_2} - C,$$

(1) with

(2) or

whereupon $\beta > 2$, $1 > \alpha_1 > 0$, $1 > \alpha_2 > 0$. Thus, function H (s) is represented in the form of $A(s-s_*)^n$, where $n = \beta + \alpha_1$ or $n = \beta - \alpha_2$.

If we as self-similar variable select the $\xi = x/a_0\sqrt{t}$, where of the $a_0^2 = Aa^2$, equation (VI.4.8) is reduced to the form:

$$\frac{d^2\sigma^n}{d\xi^2} + \frac{\xi}{2}\frac{d\sigma}{d\xi} = 0, \quad \sigma = s - s_*. \qquad \text{(VI.4.9)}$$

. This equation is related to the type, examined in chapter IV.

Let us examine separately three possible versions of the initial conditions: $s_0 = s_*$, $s_0 < s_*$ and $s_0 > s_*$. . Let first $s_0 = s_*$. As shown above (see chapter IV), if n > 1, the solutions to equation (VI.4.9) become zero at certain finite value $\xi = c$, i.e., there is a "front of impregnation" whose rate is final. In the low values of $s - s_*$ and $= c - \xi$ the solution to equation (VI.4.9) asymptotically is represented in the form:

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THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS (TEORIY--ETC(U)
JAN 77 G I BARENBLATT, V M YENTOV, V M RYZHIK

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2 OF 5

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$$c-\xi=\int_0^{s-s_*}\frac{n\sigma^{n-1}}{c_1+\frac{c}{2}\sigma}\,d\sigma. \qquad \text{(VI.4.10)}$$

In order to determine the constant $c_1$, let us require additionally in order that with $s\to s_*$ and $c-\xi\to 0$ would remain final the relation of $u_1/m(s-s_*)$, where $u_1$ is rate of filtration of liquid. This relation is the average particle speed of of the movable (continuous) part of the being absorbed liquid at front at the torque/moment of its merging/coalescence with motionless liquid, which is located in front of front.

From formula (VI.4.4) we have

$$u_1=\frac{am}{\sqrt{t}}H'(s)\frac{ds}{d\xi}. \qquad \text{(VI.4.11)}$$

566

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Page 175.

Then from (VI.4.10) it follows

$$\frac{u_1}{m(s-s_*)} = -\frac{a}{\sqrt{l}}\left(\frac{c_1}{s-s_*}+\frac{c}{2}\right).$$

Consequently, in order that the relation of $u_1/m(s-s_*)$ would

be final, it is necessary, in order to $c_1 = 0$. Then (VI.4.10) leads to asymptotic expression

$$c-\xi=\frac{2n}{c(n-1)}(s-s_*)^{n-1},\quad s-s_*=\left(\frac{c}{2}\frac{n-1}{n}\right)^{\frac{1}{n-1}}(c-\xi)^{\frac{1}{n-1}}. \tag{VI.4.12}$$

Analogous with self-similar problem for the filtration of gas the solution of problem in the case of $s_0 = s_*$ we will search for in the form of a series

$$s-s_*=\left(\frac{c}{2}\frac{n-1}{n}\right)^{\frac{1}{n-1}}\eta^{\frac{1}{n-1}}(1+a_1\eta+a_2\eta^2+\ldots). \tag{VI.4.13}$$

The coefficients of a series of $a_i$ of rated value of c are located by means of substitution into equation (VI.4.9). Changing c, we will obtain different values $s_1 = s(0)$.

We will be turned to the case wher $s_0 < s_*$. In this case,, obviously, the front of impregnation can be spread only at the final

speed. Since behind front the displacing phase is everywhere movable, at the front must be of $s = s_*$ and from $s_*$ to $s_0$ appears the jump of saturation, analogous to shock on leading edge of stabilized zone described in §3. On jump must be made the condition (VI.2.21). In this case, the $u_1^{(1)}$ is determined from the formulas (VI.4.10) and (VI.4.11) with $s = s_0,\ u_1^{(2)} = 0,$ a the speed of jump of V it will be located from the condition of $x_c = ca_0\sqrt{t},$ whence

$$V = \frac{dx_c}{dt} = \frac{ca_0}{2\sqrt{t}}, \quad u_1 = \frac{a_0 m}{\sqrt{t}} \left. \frac{c_1 + \frac{c}{2}(s - s_*)}{m(s_* - s_0)} \right|_{s \to s_*} = \frac{a_0}{\sqrt{t}} \frac{c_1}{s_* - s_0}.$$

Thus, from (VI.2.21) we have

$$\frac{ca_0}{2\sqrt{t}} = \frac{a_0}{\sqrt{t}} \frac{c_1}{s_* - s_0} \overset{(1)}{\text{или}} \quad c_1 = \frac{c}{2}(s_* - s_0). \qquad \text{(VI.4.14)}$$

(1) or

With $s_0 = s_*$ again we have condition $c_1$ = as 0, equivalent to the condition of the final speed of the motion of the being absorbed liquid.

If $s_* - s_0$, and, consequently, $c_1$ are not equal to zero, resolution (VI.4.13) takes the form:

$$s - s_* = (c_1\eta)^{\frac{1}{n}}(1 + a_1'\eta + a_2'\eta^2 + \ldots). \qquad \text{(VI.4.15)}$$

The emergence of the jump of saturation in the solution of the problem of capillary impregnation is connected with the made in 2 assumption about the fact that at any point in time the liquid in each point of the porous medium can be located only in one of the two end states - completely bound and movable or completely incoherent and therefore motionless. This leads to the unique dependence of relative permeability on the saturation of the characteristic form, depicted on Fig. VI.5, with the point of the $s_*$, where of the $f_1(s_*) = 0$, a with $s < s_*$ liquid is motionless. More detailed investigation shows that actually only the part of the liquid is located in each of the states, whereupon between connected and incoherent parts occurs the exchange of liquid before the achievement of certain equilibrium distribution. In this case the jump of saturation at the front of capillary impregnation is replaced by the narrow zone of the smooth transition from $s_0$ to of $s_*$.

As for the filtration of gas, the final velocity of propagation of front makes it possible to utilize the obtained solution not only for infinite, but also for finite domains, to the approach of front to the moved away end/lead.

Let now $s_0 > s_*$. Equation (VI.4.12) can be rewritten in the form:

$$n(s-s_*)^{n-1}\frac{d^2v}{d\xi^2}+\frac{\xi}{2}\frac{dv}{d\xi}=0 \qquad v=(s-s_*)^n. \qquad \text{(VI.4.16)}$$

If $s_0 > s_*$, equation (VI.4.16) does not have special feature/peculiarities, since the coefficient of higher derivative does not become zero. Furthermore, from the common properties of parabolic equations it follows that s is changed monotonically from $s_1$ to $s_0$ during change $\xi$ from 0 to $\infty$. Therefore $v - v_0$ and, consequently, also s are $s_0$ cannot become zero into which end point and $s \to s_0$ only asymptotically with $\xi \to \infty$. Solution for this case can be obtained numerically, by being given at this value $s_1 = s(0)$

certain value $ds/d\xi$ ($\xi$ = 0). Then, solving the problem of Cauchy, we obtain the solution, which corresponds to the determined value $s_0 = s$ (*). By changing $ds/d\xi$, it is possible to find the solution, which corresponds to rated value $s_0$.

Dependence $s_1$ (c) with $s_0 \leqslant s_*$ it is possible to find in an explicit form, utilizing that easily checked fact, that if certain function of $-s_* = \varphi_0(\xi)$ satisfies an equation (VI.4.9), then function

$$s - s_* = \varphi(\xi) = c^{\frac{2}{n-1}} \varphi_0\left(\frac{\xi}{c}\right) \qquad \text{(VI.4.17)}$$

also will be the solution to this equation. Let the $\varphi_0(\xi)$ is a solution to equation (VI.4.9) such, that $\varphi_0(1) = 0$, represented by formula (VI.4.13) or (VI.4.15) for c = 1.

Condition (VI.4.14) will be observed, also, for all functions of the $\varphi(\xi)$, expressed by formula (VI.4.17). Thus, by knowing the solution to stated problem in certain value $s_1$ or c, it is possible

to obtain all the solutions for that which was assigned $s_0$. From formula (VI.4.17) we will obtain communication/connection between $s_1$ and c in the form:

$$s_1 = c^{\frac{2}{n-1}} \varphi_0(0) \text{ или } c = \left( \frac{s_1}{\varphi_0(0)} \right)^{\frac{n-1}{2}}. \qquad \text{(VI.4.18)}$$

Page 177.

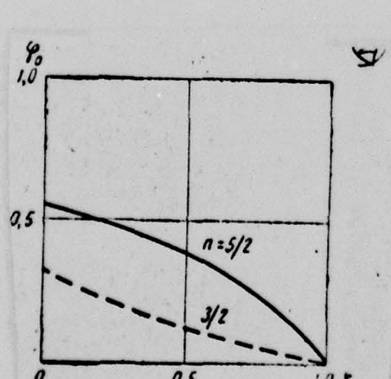



Fig. VI.15.

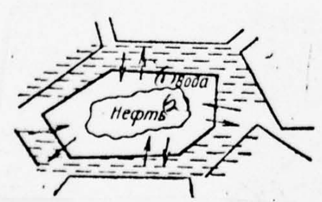



Fig. VI.16.

Key: (1). Water. (2). Oil.

Fig. VI.17.

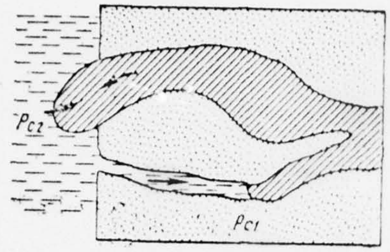


Figure VI.15 depicts the calculated with the aid of a series (VI.4.13) curved $\varphi_0(\xi)$ for n = 3/2 and n = 5/2.

The obtained solution can be utilized for determining the parameters of the porous medium from the results of the experimental measurement of the velocity of capillary impregnation. If the velocity of the "front of impregnation" is final, then this can be made, by measuring the coordinates of the front of $x_c$ at different points in time.

As a result of the self-similarity of the problem of $x_c$ it is expressed according to the formula of $x_c = ca_0\sqrt{t}$.

By using relationship/ratio (VI.4.18), $x_c$ can be expressed in the form:

$$x_c = N s_1^{\frac{n-1}{2}} \left(\frac{\alpha}{\mu_1} \sqrt{\frac{k}{m}}\, t\right)^{1/2}, \tag{VI.4.19}$$

where the $N = A^{\frac{1}{2}} \left[\varphi_0 (0)\right]^{-\frac{n-1}{2}}$.

By measuring experimentally the relaticn cf aaaaaaa and value $s_1$, it is possible to find exponent n and ccefficient N, which characterizes the structure of pore space. The experiments of this type are described in wcrk [77].

2. Ancther process, in which the filtration occurs under the action only of capillary forces, ccuntercurrent capillary impregnaticn, appears in such a case, code the section of the porous medium, engaged by the less wetting phase, it proves to be the completely surrounded another, more wetting liquid (Fig. VI.16).

Under such conditions the more wetting liquid for pin-head blisters is absorbed into specimen/sample, displacing the less wetting phase on adjacent large pores. Simpler anything the mechanism of this phenomenon is illustrated in an example of two adjacent pore channels, connected with end/leads (Fig. VI.17).

Page 178.

Countercurrent impregnation appears, for example, in the oil strata, broken by cracks into separate blocks (cracked layers), when moving water faster moves over cracks and blocks prove to be surrounded water. After this the water is absorbed into blocks, and petroleum emerges from them through the same surface.

Investigate countercurrent capillary impregnation for the linear case. Let us examine as in the preceding/previous point/item, the cylindrical specimen/sample of the porous medium, lateral surfaces of which are impenetrable. Initially specimen/sample is filled by the nonwetting phase. If one of the end/faces of cylinder also is inpenetrable, and another is led to contact with the wetting liquid, then will be initiated countercurrent capillary impregnation. This

means that the wetting phase will be absorbed, and not wetting emerge through the sole open face. The described linear specimen/sample can be considered as element of the assembly of more complex form, for example rectangular.

As show experiments in the countercurrent capillary impregnation of transparent specimen/samples, filtration in contrary direction occurs evenly over entire section, i.e., "channels", over which moves each of the phases as during usual two-phase flow, are comparable in size/dimensions with the diameter of pores. Therefore countercurrent filtration can be examined within the framework of the representations, accepted for ramjet/direct-flow two-phase filtration, and record/written the law of filtration in the form:

$$u_i = -\frac{k}{\mu_i} f_i^*(s) \frac{\partial p_i}{\partial x}; \quad p_2 - p_1 = p_c^*(s). \qquad (i = 1, 2). \qquad \text{(VI.4.20)}$$

One should only consider that, until now, relative permeability we examined only for the case when both phases move to one side. The countercurrent motion of phases will influence, of course, for phase distribution in pores, and the form of the curves

of relative permeability and capillary pressure will change. At present there are no direct/straight experimental data on the curves of relative permeability during countercurrent motion. For the qualitative investigation we will accept them the same, as before i.e. let us accept $f_i^*(s) = f_i(s)$, $p_c^*(s) = p_c(s)$.

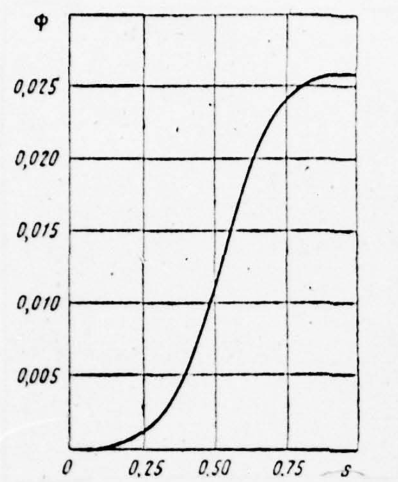
The equations of continuity for each of the phases retain the usual form:

$$m\frac{\partial s}{\partial t}+\frac{\partial u_1}{\partial x}=0;\quad m\frac{\partial s}{\partial t}-\frac{\partial u_2}{\partial x}=0. \qquad \text{(VI.4.21)}$$

For countercurrent motion from equations (VI.4.21) it follows

$$u_1+u_2=0. \qquad \text{(VI.4.22)}$$

Fig. VI.18.



Eliminating $dp_t/dx$ from system (VI.4.20) - (VI.4.22), we obtain

$$u_1 = -u_2 = -a^2 m \frac{\partial \Phi}{\partial x} \qquad \text{(VI.4.23)}$$

and

$$\frac{\partial s}{\partial t} - a^2 \frac{\partial^2 \Phi}{\partial x^2} = 0\,, \qquad \text{(VI.4.24)}$$

where

$$\Phi(s) = -\int_0^s f_2(s) F(s) J^*(s)\, ds, \quad a^2 = \frac{\alpha}{\mu_0} \sqrt{\frac{k}{m}}\,.$$

The equation (VI.4.24) is a special case of the equation of rapoport - Lis with w = 0. This equaticn ccincides in form with

equation (VI.4.6), the only function H (s) is replaced by $\Phi(s)$. Boundary conditions are also identical for the capillary impregnation of the gas-saturated specimen/sample and for a countercurrent capillary impregnation, i.e., the condition of equality to zero of capillary pressure at entrance ($s_1 = s^*$, $J(s^*) = 0$) and equality to zero of expenditure/consumption at enclosed end/lead. And in exactly the same manner the problem of impregnation becomes self-similar, if enclosed end/lead is infiritely moved away and the initial saturation is constant along specimen/sample. Under these conditions it is possible instead of x and t to introduce self-similar alternating/variable $\xi = x/a\sqrt{t}$.

Equation for a saturation in the case of impregnation takes in self-similar variables the following form:

$$\frac{d^2\Phi}{d\xi^2} + \frac{\xi}{2}\frac{ds}{d\xi} = 0, \qquad \text{(VI.4.25)}$$

i.e. coincides with equation (VI.4.8). Assuming that the qualitatively curves of relative permeability for a countercurrent are such as during the equally directed filtration, will use the same

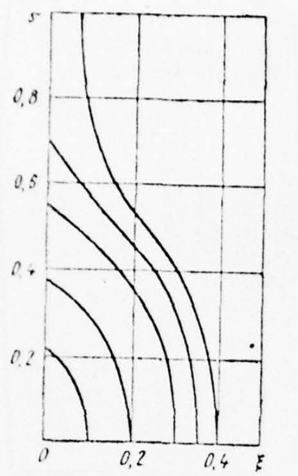
representations for $f_1$ (s) and J (s), as in the preceding/previous point/item (i.e. $f_1(s) = b(s - s_*)^\beta$, $J(s) = B(s - s_*)^{-\gamma} + C$), we will obtain that at small $s - s_*$ the function of $\Phi$ is represented in the form:

$$\Phi(s) \approx N(s - s_*)^n \qquad n > 1$$

($f_2$ (s) remains not equal to zero). The typical form of the function of $\Phi(s)$ is shown in Fig. VI.18.

Thus, with small $s - s_*$ equation (VI.4.25) coincides in form with equation (VI.4.9). Therefore the qualitative conclusion/derivations about the character of the solutions to equation (VI.4.9) in the different values of the initial saturation $s_0$, including the conclusion/derivation about the final velocity of the "front of impregnation" in $s_0 \leqslant s_*$ and conditions on the jump in of $s_0 < s_*$ are retained for the solution to equation (VI.4.25). Specifically, since the rate of filtration of the first phase is expressed by formula (VI.4.23), are retained asymptotic expressions (VI.4.13) and (VI.4.18) for the low values of $s - s_*$.

Fig. VI.19.




~~Fig. VI.20.~~

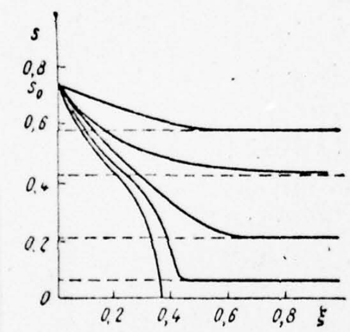



Fig VI. 20

The solution to equation (VI.4.25) possible, as earlier, to find, being given value $\xi = c$ by such, in order to $s(c) = s_*$. Near point (c, 0) this solution can be presented in the form of a series (VI.4.16) or (VI.4.18), and at great significance of s - found any numerical method. The solution for assigned $s_1 = s(0) = s^*$ is determined by selection by change c, since the required accuracy is small.

If $s_0$ is more the motionless saturation of $s_*$, that as for the problem of preceding/previous paragraph, $s = s_0$ only with $\xi \to \infty$. Family of solutions is located by the numerical solution of the Cauchy problem in assigned $s_1 = s(0)$ with the different values of $ds/d\xi|_{\xi=0}$. To each of these solutions corresponds certain value $s_0$. The unknown solution for that which was assigned $s_1 = s(0)$ again is determined by selection.

As an example Fig. VI.19 and VI.20 give the curves of $s(\xi)$, found at the following selection of relative permeability and function of the Leverett:

$$f_1 = s^4;\quad f_2 = (1+s)(1-s)^3;\quad J(s) = s^{-1/2} - 1;$$

$\mu_1/\mu_2 = \mu_0$ was accepted equal to 1. Figure VI.19 shows curves, that correspond to case $s_0$ = to 0 and different c. It is evident that with $c > 0.38$ curves of s ($\xi$) secant s = 1 with $\xi > 0$. Consequently, the physical sense they have the only those curves for which $c < 0.38$.

Fig. VI.21.

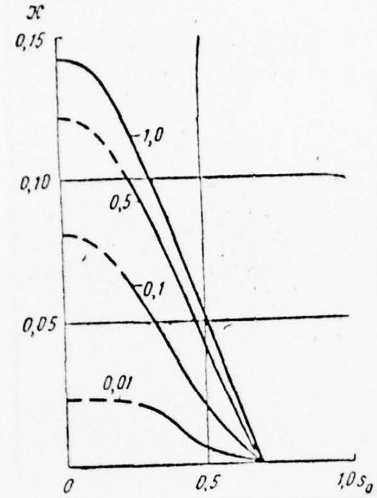


Figure VI.20 depicts the curves of s (ξ), which correspond to the different values $s_0$ with $s_1$ = 0.75. Let us note that the amount of liquid Q, which soaked itself into the layer through the unit of sectional area up to point in time t, is equal

$$Q = ma\sqrt{t}\,K(s_1, s_0), \qquad \text{(VI.4.26)}$$

where

$$K(s_1, s_0) = \int_0^\infty (s - s_0)\, d\xi.$$

On Fig. VI.21 are constructed the curves of dependences K ($s_0$) for $s_1$= 0.7 in the different relations of viscosity ($\mu_0$ = 0.5 * 0.1 and 0.01).

In order to utilize data on countercurrent capillary impregnation in the problems of the displacement of the nonmiscible

liquids from the heterogeneous and cracked-porous media (see chapter VII 3), it is necessary to have a curve of the dependence of the average saturation of the specimen/sample of the finite length on time. If the initial saturation of specimen/sample $s_0$ incoherent or is equal to zero, the rate of the "front of impregnation" is final. Therefore solution for the specimen/sample of the finite length up to approach to enclosed end/lead coincides with self-similar. In this case, from formula (VI.4.26) it follows that the average saturation of specimen/sample is expressed by formula

$$\bar{s} = s_0 + \frac{a}{l} \sqrt{t}\, K(s_1, s_0). \qquad \text{(VI.4.27)}$$

In order to obtain the dependence of aaaa on t for the subsequent points in time, one should solve partial differential equation (VI.4.24). For the qualitative investigation we will be restricted to the fact that let us find approximate solution. Let us integrate equation (VI.4.24) for x from 0 to $l$, assuming that the front of impregnation already it approached to enclosed end/lead and $s(l) > s_*$. Then, utilizing boundary conditions, we have

$$a^2\Phi'(s_1)\left(\frac{\partial s}{\partial x}\right)_0 = \frac{d}{dt}\int_0^l s\,dx. \qquad \text{(VI.4.28)}$$

Taking into account the condition of $d\bar{s}/dx = 0$ with $x = l$, we will search for distribution s in the form:

$$s = s_1 - A(t)(2l - x)x. \quad \text{(VI.4.29)}$$

Page 182.

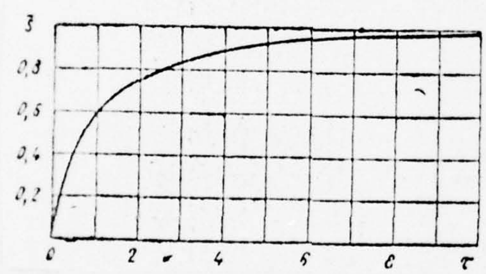



Fig. VI.22.

The average saturation of $\bar{s}$ is equal to

$$\bar{s}=\frac{1}{l}\int_0^l s\,dx=s_1-\frac{2}{3}A(t)\,l^2. \qquad \text{(VI.4.30)}$$

. Therefore

$$s=s_1-\frac{2}{3}(s_1-\bar{s})\left(2-\frac{x}{l}\right)\frac{x}{l}. \qquad \text{(VI.4.31)}$$

By substituting in the equation (VI.4.28) of the expression of $\left(\frac{\partial s}{\partial x}\right)_0$ and $\int_0^l s dx$, those which were obtained from formula (VI.4.31), let us find

$$\frac{d\bar{s}}{dt}=\frac{3a^2}{l^2}\Phi'(s_1)(s_1-\bar{s}). \qquad \text{(VI.4.32)}$$

Equation (VI.4.32) must be solved under the initial condition,

which expresses the continuity of the average saturation at the torque/moment of the apprcach of front toward the end of the specimen/sample: with the $t=t_0$ $\bar{s}=\bar{s}_0$, where cf the $\bar{s}_0$ it is cbtained from formula (VI.4.27), i.e.,

$$\bar{s}_0 = s_0 + K/c,\ t_0 = l^2/c^2a^2.$$

The unknown solution takes the fcrm:

$$\bar{s} = s_1 - \left(s_1 - s_0 - \frac{K}{c}\right)\exp\left[-3\Phi'(s_1)\left(\frac{a^2t}{l^2} - \frac{1}{c^2}\right)\right]. \quad \text{(VI.4.33)}$$

Thus, with $t < t_0$ the average saturation is expressed by the formula (VI.4.27), and with $t > t_0$ - by formula (VI.4.33). The general view of the dependence of the $\bar{s}(\tau)$, where of the $\tau = \frac{a^2t}{l^2}$, is shown in Fig. VI.22 for the conditicrs cf the examined example.

3. In the designations, used in p. 2, the equation of rapoport -

Lis (VI.3.4) signs the form:

$$\frac{\partial s}{\partial t}+\frac{u}{m}F'(s)\frac{\partial s}{\partial x}-a^2\frac{\partial^2\Phi}{\partial x^2}=0, \qquad \text{(VI.4.34)}$$

in this case for the rates cf filtration cf each of the phases we have in accordance with (VI.3.17)

$$u_1=uF(s)-a^2m\Phi'(s)\frac{ds}{dx}, \quad u_1+u_2=u(t). \qquad \text{(VI.4.35)}$$

Page 183.

Total velocity u (t) is a function of time but must be assigned cr determined from boundary conditions for a pressure. If we dependence u (t) assign in the form of $u=\frac{q_0}{\sqrt{t}}$, that equation

(VI.3.34) has the self-similar solution of the form of $s = s(\xi),\ \xi = x/a\sqrt{t}$. For the existence of self-similar solution, it is required also, in order to s (x, 0) = $s_0$ = const and $u_2$ (0, t) = 0. Then we obtain instead of (VI.4.34) equation

$$\frac{d^2\Phi}{d\xi^2} + \left[\frac{\xi}{2} + \lambda F'(s)\right]\frac{ds}{d\xi} = 0 \qquad \left(\lambda = \frac{q_0}{a}\right). \qquad \text{(VI.4.36)}$$

. With $\xi$ = 0 must be made condition (equivalent to condition $u_2$ = 0)

$$\frac{ds}{d\xi} = \frac{\lambda\mu_0}{f_1(s)\, f'(s)}. \qquad \text{(VI.4.37)}$$

. This follows from formulas (VI.4.24) and (VI.4.35). Furthermore, with $\xi \to \infty$ $s \to s_0$. Near front, i.e., with s, close to $s_0$, the equation (VI.4.36) asymptotically transfer/converts to equation (VI.4.25), since $\xi \gg 2\lambda F'(s)$, either because $\xi$ very greatly (with $s_0 > s_*$), or because $F'(s) \to 0$ (with $s_0 \leqslant s_*$). In connection with this the conditions at the front of displacement at different values

$s_0$ are retained the same as and in the case of capillary impregnation. With $s_0 \leqslant s_*$ the velocity of front is final, and for a numerical count with small $\eta = c - \xi$ it is possible to utilize expansions (VI.4.13) or (VI.4.18). Satisfaction of condition (VI.4.37) it is possible to attain, by changing value $\xi = c$ with which s turns into $s_*$.

As an example Fig. VI.23 gives the solutions to equation (VI.4.36), that satisfy condition (VI.4.37) in $\lambda = 0.25$ and $\lambda = 1.0$. Functions $f_1$ (s) and J (s) were used the same as in an example in p. 2. It is characteristic that during a change of the initial saturation $s_0$ in very wide limits the final saturation $s_1$ is changed insignificantly. So, with $\lambda = 0.25$ to change $s_0$ from 0 to 0.90 corresponds change $s_1$ from 0.89 to 0.93. In the case $\lambda = 1$ change $s_1$ is still less: $0.992 < s_1 < 0.998$ with $0 < s_0 < 0.98$.

The obtained solution of the problem of the displacement of the nonmiscible liquids taking into account capillary forces is interesting to compare with the solution of the same problem in the setting of Bakley-Leverett. In the designations of the present paragraph, the solution of Bakley-Leverett (VI.2.21) takes the form:

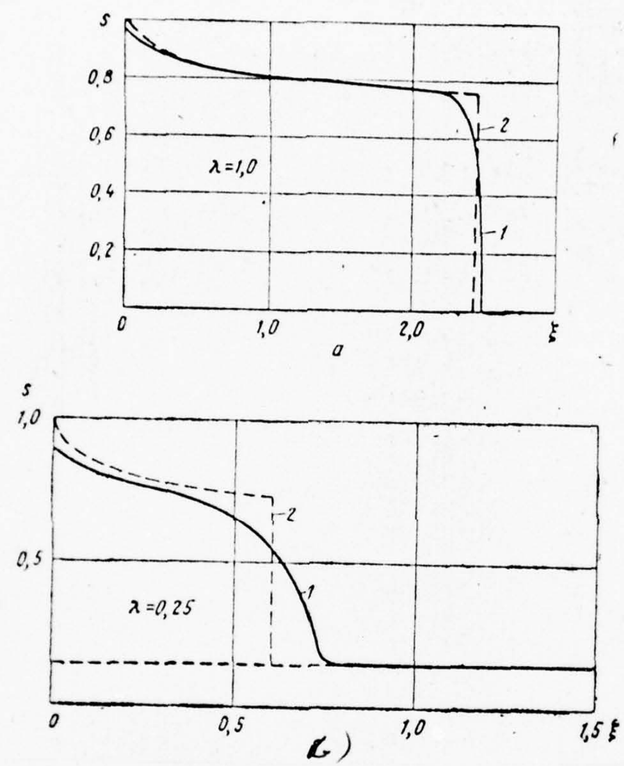
$$\xi = 2\lambda F'(s) \tag{VI.4.38}$$

(since at the moment with $\xi \to \infty$ ($t = 0$) $s = s_0$ = const). With in the solution of Bakley-Leverett is introduced the jump of saturation whose position can be found from formula

$$\xi_c = 2\lambda F'(s_0) = 2\lambda \frac{F(s_c) - F(s_0)}{s_c - s_0}. \tag{VI.4.39}$$

According to formulas (VI.4.39) it is possible to find $s_c$ and $\xi_c$, by knowing $s_0$.

Fig. VI.23.



In Fig. VI.23a, b together with the solutions to equation (VI.4.36) (is curve 1) are brought also curved 2 for s (ξ), calculated from formula (VI.4.38) on condition (VI.4.39) for the same values λ (i.e. λ = 1 and λ = 0.25). From the given curves it is evident that in λ = 1 solution of Bakley-Leverett is very close to precise. Specifically, for the solution of Bakley-Leverett of $\xi(s_c) = 2{,}45$, a for exact solution of c = 2.49. In the case λ = 0.25 disagreement is more essential.

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Pages 185-216.

Chapter VII.

Unsteady filtration in [redacted] cracked-porous MATERIAL, MATERIAL [redacted] with dual porosity and STRATIFIED [redacted] rocks.

§1. Filtration of homogeneous liquid in the cracked-porous medium.

Along with the granulated porous media in which the liquid is contained and it moves in with inter-grain space, are encountered also the cracked minerals in which there is the developed system of cracks, in full or in part cause of filtration properties of the medium. The importance of research on such media is determined by the fact that a series of the greatest deposits of oil is timed to the rock/species in which there are numerous cracks.

The specific character of the cracked medium is caused by the fact that crack - this (schematically) the narrow slot, two measurements of which thousand times are more the third, unlike the pores, all size/dimensions of which one order. As a result of this even with the most insignificant volume of cracks in common/general/total void content in solid skeleton they can have a determining effect on the character of the motion of liquid. In this, is exhibited the common for entire theory of filtration property of its objects: hydrodynamic characteristics can depend substantially on such structural constituents, statistical weight of which is negligeably small.

Usually are distinguished purely the cracked and cracked-porous media. The first of them are the blocks of the mineral, between which there are cracks, whereupon blocks themselves are impenetrable and are not interchanged with liquid with cracks (for example cracked granite); in the cracked-porous medium the blocks are the pieces of the usual porous medium, with final porosity and permeability (cracked limestone). In all cases the volume of cracks is negligible in comparison with the total volume, occupied with solid skeleton and voids; in the majority of cases it is small in comparison with the overall void content, which are composed of the volume of the pore space of porous blocks and volume of cracks themselves.

Page 186.

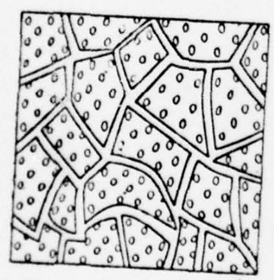


Fig. VII.1.

Only when its own porosity of blocks is virtually equal to zero (for example of the cracked igneous rocks), it is necessary to allow the volume strictly of cracks.

On the contrary, in the majority of cases the hydraulic conductivity of the system of cracks many times is more the hydraulic conductivity of blocks. Therefore it is possible to say that in the cracked-porous medium liquid "is stored" in porous blocks, and is moved on cracks. During the steady motion of liquid, this does not lead to essential differences from the usual porous medium. However, during unsteady processes and in the course of the displacement of one liquid of another is exhibited a series of special feature/peculiarities, still not studied to end/lead. Filtration in the purely cracked media occurs qualitatively just as in the usual porous media, only with small quantitative deviations. Therefore subsequently the primary attention is given to the cracked-porous media.

1. For the viscous motion of viscous fluid in slot with parallel walls, is valid Boussinesq's formula

$$Q = -\frac{bh^3}{12\mu}\frac{dp}{dx}. \qquad \text{(VII.1.1)}$$

. Here Q is a flow rate of liquid; b - the width of slot in the section, perpendicular to X-axis; h - the expansion/disclosure of slot; $\mu$ - the viscosity of liquid; p - pressure.

The existence of this simple formula of motion in various crack impelled many researchers to the searches for the expressions, describing motion in the ordered system of cracks. However, this approach turned out to be less fruitful, than the description of current in cracked-porous rock/species by the methods of the mechanics of continuous medium.

Let us assume that the cracked-porous medium consists of pulley tackle, isolated from each other by cracks, whereupon the form and the location of blocks are irregular (Fig. VII.1). Let us take as elementary macrovolume (comp. chapter I) the volume whose

size/dimensions are great in comparison with the size/dimensions of separate block and, consequently, also the which interest us processes they occur to scale considerably larger, than the size/dimension of block [1].

FOOTNOTE [1]. The size/dimensions of blocks (and, therefore, the length of cracks $l$) are by the different. The set forth approach is based on the assumption that the $d \ll l \ll L$, i.e., blocks are great in comparison with the size/dimension of pores d, but are small in comparison with the size/dimension of layer L. ENDFOOTNOTE.

Let us examine the at first most essential case when the permeability of blocks is small so, that during the description of the macroscopic motion of liquid it can be disregarded.

Page 187.

By considering motion in cracks slow (inertia-free), it is possible to write for it the law of darcy, which is derive/concluded from dimensional analysis just as in chapter I. In this case,, taking into

account the possible anisotropic system of cracks and the fact that each crack is characterized by two size/dimensions - by the length of $l$ and by expansion/disclosure h, the law of filtration is convenient to represent in the form:

$$u_i = -\frac{k_{ij}}{\mu}\frac{\partial p}{\partial x_j} \equiv -\frac{h^3}{\mu l}k^0_{ij}\frac{\partial p}{\partial x_j}. \qquad \text{(VII.1.2)}$$

. Here $u_i$ - the components of the velocity vector of filtration, determined by usual method; the symmetrical tensor of $k_{ij}$ is called the tensor of interstitial permeability; h is the average crack opening; $l$ - the significant dimension of block. The concrete/specific/actual form of the dimensionless tensor of the permeability of $k^0_{ij}$ is determined by the geometry of the system of cracks; forthe medium, which consists of impenetrable blocks and several systems of the dense regularly harranged/located cracks, it will be able to be obtained on the basis of Boussinesq's formula (VII.1.1).

In the general case of the cracked-porous medium, the law of filtration also takes the tensor form (VII.1.2). however, the

calculation of the components of the tensor of permeability in this case is impossible, and them they determine, on the strength of observational data. by the appropriate selection of the coordinate system the tensor of $k_{ij}$ can be led to the principal axes. If the vector of pressure gradient is directed along one of the principal axes, then the velocity vector of filtration is directed then.

2. As already it was mentioned, the characteristic feature cracked-porous medium lies in the fact that the motion of liquid in this medium occurs in essence on cracks, while the volume of cracks is small, and the main supplies of liquid they entail porous blocks. Let us assume that we disregarded the motion of liquid in blocks, and on the boundary of the cracked-porous layer liquid in which initially was located $P_0$, occurs a decompression to certain other value $P_1$. By disregarding the permeability of blocks, it is possible to utilize for the description of motion in cracks usual relationship/ratios of filtration theory in the porous medium (for example in the case of slightly-compressible liquid and elastic-deformed layer - by the relationship/ratios of the theory of elastic mode/conditions). After certain transient process in cracks, will be establish/installed the new stationary distribution of pressure, whereupon at least near the boundary of layer pressure will prove to be considerably lower than initial. Since the pressure in blocks on the strength of their

assumed impenetrability could not change, between the liquid in blocks and the liquid in cracks is created a considerable pressure difference - order $P_0-P_1$, and consequently, but blocks appear the local gradients of the pressures of $(P_0-P_1)/l$, which considerably exceed the existing in layer pressure gradient in the cracks of $\sim(P_0-P_1)/L$. Under these conditions in layer even with most insignificant permeability of blocks appear the local filtration flows, which cause the inflow of liquid from blocks into cracks and the equalization of the local pressure gradients between blocks and cracks.

Page 188.

The fact that in the cracked-porous medium can in unsteady process appear the local pressure differences and the local return flows between blocks and cracks, we will place as the basis of the description of the cracked-porous medium, which consists of low-penetrability porous blocks and the cracks, total volume of which is small.

Let us introduce instead of one mean pressure of liquid at the

particular point of the medium two pressures - pressure in cracks $p_1$ and pressure in pores of blocks $p_2$. On the assumption that the permeability of blocks $k_2$ is very small, we can for determining filtration fluid flow through certain area/site of the medium utilize an equation (VII.1.2), substituting in it the value of pressure in cracks $p_1$.

Let us compose now the equations of the balance of liquid in cracks and blocks. Designating through $m_1$ interstitial porosity (ratio of the volume of cracks to the complete volume of the medium), have

$$\frac{\partial (m_1\rho)}{\partial t} + \operatorname{div}(\rho\vec{u}) - q = 0, \qquad \text{(VII.1.3)}$$

where q is an amount of liquid, which overflows for time unit from blocks into cracks per unit of volume of the medium.

For blocks it is possible to disregard direct filtration flow, so that the equation of continuity takes the form:

$$\frac{\partial (m_2\rho)}{\partial t} + q = 0, \qquad \text{(VII.1.4)}$$

where $m_2$ is a porosity of blocks (taking into account the total volume of the medium).

In order to lock the obtained system of equations, it is necessary, besides the equation of state of liquids and equations, which connect the change in porosity $m_1$ and $m_2$ with pressure, to give expression for flow q. This expression can be obtained from dimensional analysis. Let us note first of all, that since the motion of liquid in layer is considered inertia-free, inertia-free must be the movement of liquid in blocks. Further flow q can depend on pressure in blocks $p_2$ and in cracks $p_1$, the size/dimension of blocks $l_1$, the permeability of blocks $k_2$, of the viscosity of liquid $\mu$, of its density $\rho$ must become zero with the equality of pressures $p_1$ and $p_2$. Let us assume at first that the density $\rho$ and viscosity $\mu$ liquid barely depend on pressure in interval/gap $p_1<p<p_2$ and them it is possible to consider constants, just as the permeability of blocks

$k_2$. Then expression for q must be invariant relative to the selection of the reference point of pressure can depend crly on difference $p_1 - p_2$. Thus, value q depends on the dimensicnal values $p_1 - p_2$, $\rho$, $\mu$ $k_2$, $l$.

Let us note now that as a result cf the noninertia of the motion of dimensicnality of permeability, pressure and the viscosity can be selected independently, under one condition alcne

$$[k_2][p][\mu]^{-1} = L^2 T^{-1};$$

besides this it is possible to count that the dimensionality of mass M is not connected with the dimensionality cf pressure or viscosity.

Page 189.

Hence follows

$$q = \alpha \frac{\rho k_2}{\mu} \frac{p_2 - p_1}{l^2}, \qquad \text{(VII.1.5)}$$

where $\alpha$ - dimensionless distance that characterizes the geometry of the medium. relationship/ratio (VII.1.5) must be refined if the density of liquid $\rho$ and its viscosity $\mu$ depend on pressure. Assuming that the law of filtration in blocks can be represented in the form:

$$\rho u_i = -\frac{k_2 \rho_0}{\mu_0} \frac{\partial f(p)}{\partial x_i},$$

where $\rho_0$ and $\mu_0$ are characteristic constant values $\rho$ and $\mu$, but f (p) - the function of the dimensionality of pressure, relationship/ratio (VII.1.5) can be rewritten thus:

$$q = \frac{\alpha \rho_0 k_2}{l^2} \frac{f(p_2) - f(p_1)}{\mu_0}. \qquad \text{(VII.1.6)}$$

For example during the filtration of thermodynamically perfect gas $f = p^2/2p_0$, and expression (VII.1.6) gives

$$q = \frac{\alpha \rho_0 k_2}{2l^2 p_0^2 \mu}(p_2^2 - p_1^2), \qquad \text{(VII.1.7)}$$

where $p_0$ - the pressure, which corresponds density $\rho_0$.

The interstitial porosity $m_1$ is usually small and by it in the majority of cases it is possible to disregard if the medium is cracked-porous (but not by purely cracked), but the porosity of blocks $m_2$ to consider the function of both pressures $p_1$ and $p_2$. Being limited to linear approximation, it has a relationship/ratio

$$\frac{\partial m_2}{\partial t} = m_{20}\left(\beta_{21}\frac{\partial p_1}{\partial t} + \beta_{22}\frac{\partial p_2}{\partial t}\right), \qquad \text{(VII.1.8)}$$

where the values of $\beta_{21}$, $\beta_{22}$ and $m_{20}$ at small changes of porosity can be considered constants.

A change in the porosity, as usual, one should consider only in those expressions where the porosity is differentiated; furthermore, since it enters in product with the value of the density of liquid $\rho$, of a change of the porosity are essential only in the case slightly-compressible (drop) liquid; during the filtration of gas changes in the porosity can be disregarded. Being limited by the case of true liquid, we have

$$\rho = \rho_0[1 + \beta_*(p - p_0)], \qquad \text{(VII.1.9)}$$

where p = $p_1$, $p_2$, depending on whether is examined liquid in cracks or in blocks.

Page 190.

Substituting expressions (VII.1.2), (VII.1.8) and (VII.1.9) in equations (VII.1.3) and (VII.1.4) and set/assuming $m_1$ = 0, we have a system of equations

$$\frac{\rho_0}{\mu}\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial p_1}{\partial x_j}\right)-\frac{\alpha\rho_0 k_2}{l^2}\frac{p_2-p_1}{\mu}=0;$$

$$m_0\rho_0\left[-\beta_{21}\frac{\partial p_1}{\partial t}+(\beta_{22}+\beta_*)\frac{\partial p_2}{\partial t}\right]+\frac{\alpha\rho_0 k_2}{l^2}\frac{p_2-p_1}{\mu}=0. \quad \text{(VII.1.10)}$$

Most frequently is examined the case when the medium is uniform and isotropic and interstitial permeability it is expressed by the isotropic component of a tensor of $k_{ij}=k_1\delta_{ij}$. In this case, the system (VII.1.10) accepts the simple form:

$$\frac{\partial p_2}{\partial t} - \beta \frac{\partial p_1}{\partial t} + A(p_2 - p_1) = 0;$$
$$\varkappa \nabla^2 p_1 - A(p_2 - p_1) = 0, \qquad \text{(VII.1.11)}$$

where

$$A = \frac{\alpha k_2}{\mu l^2 m_0 (\beta_{22} + \beta_*)}; \quad \varkappa = \frac{k_1}{\mu m_0 (\beta_{22} + \beta_*)}; \quad \beta = \frac{\beta_{21}}{\beta_{22} + \beta_*}.$$

From system (VII.1.11) it is possible tc exclude one of the pressures; after determining from the second equation $p_2$ and after substituting the obtained value into the first equation, we have

$$\frac{\partial p_1}{\partial t} - \eta \frac{\partial \nabla^2 p_1}{\partial t} = \frac{\varkappa}{1-\beta} \nabla^2 p_1;$$

$$\eta = \frac{\varkappa}{A(1-\beta)} = \frac{k_1 l^2}{\alpha k_2 (1-\beta)}. \qquad \text{(VII.1.12)}$$

In the limit with of $\eta \to 0$, which corresponds to the unimpeded exchange of the liquid between by blocks and cracks, equation (VII.1.12) transfer/converts to the usual equation of elastic mode/conditions with the coefficient of the piezoconductivity of $\varkappa/(1-\beta)$; it is not difficult to see that this coefficient of piezoconductivity answers the permeability of system of cracks, but porosity and the compressibility of blocks.

3. Equation (VI.1.12) and system (VII.1.11) possess a series of the special feature/peculiarities which at first glance seem uncommon

and reason for which lie/rests at the degenerate character of the system (VII.1.11), which relates to the medium with negligible by interstitial porosity and the permeability of blocks. In connection with this is of interest the study of the properties of the solutions of this system.

Let us note that the equation of form (VII.1.12) satisfies not only pressure $p_1$, but also pressure $p_2$ and, therefore, any linear combination of these pressures. In order to be convinced of this, is sufficient the second equation (VII.1.12) to multiply on $\beta/A$ and to differentiate for t, and then to add to the initial equation. After this of the system (VII.1.11) easily is eliminated $p_1$. This shows that in both pressures and any combination of them are inherent those property which must possess any solution to equation (VII.1.12) (see below). At the same time, as it is easy to be convinced, not all these linear combinations are equal.

Page 191.

Among them there is one, namely $p = p_2 - \beta p_1$, which must be continuous on time in the locked domain of definition of solution, including

boundary t = 0. Actually, let necessary to find limited solution of system of equations (VII.1.11) in spatial domain D in $0 \leqslant t \leqslant T$: are assigned the initial distributions of pressures $p_1$ and $p_2$. Integrating the first equation (VII.1.11) for a small interval of time of $0 \leqslant t \leqslant \varepsilon$ and fixing $\varepsilon$ to zero, we find $\lim_{t \to 0} p(x, t) = p(x, 0)$. Let us present now the second equation of system (VII.1.11) in the form:

$$-Ap + (1-\beta) Ap_1 + \varkappa \nabla^2 p_1 = 0.$$

If we select sufficiently short points in time, then the first term of this expression will approach its initial value of p (x, 0). Consequently, to the same value with opposite sign will strive the sum of other two terms. Therefore for that, in order to pressure $p_1$ (x, t) it was continuous with $t \to 0$, necessary, in order to the initial distribution $p_1$ (x, 0) it satisfied an equation

$$\varkappa \nabla^2 p_1 + (1-\beta) A p_1 = A p(x, 0) \qquad \text{(VII.1.13)}$$

under the appropriate boundary conditions. Otherwise the pressure in cracks $p_1$ (x, t) with t = 0, abruptly changes in accordance with equation (VII.1.13). In this case,, if $\beta \neq 0$ and therefore p $\neq$ $p_2$, occurs also the instantaneous redistribution of pressure in pores $p_2$ at the constant/invariable pressure p.

This behavior of solution makes simple physical sense. A change in pressures $p_1$ and $p_2$ produces change in the mass of the liquid, which fills porous blocks. Any this change leads to the return flow of certain amount of liquid from blocks to cracks or conversely. If a change in the mass of liquid certainly (not it is infinitely small), it requires finite time, since it occurs under the action limiting forces of pressure which cannot cause the infinite speeds of return flow. This shows that an instantaneous change of the mass of the included in blocks liquid is impossible, and consequently, it is impossible and instantaneous change in reduced pressure $p = p_2 - \beta p_1$,

identical to that connected with this mass. But if pressures $p_1$ and $p_2$ simultaneously change with jump in such a way that reduced pressure $p$ is not changed, then the displacements of liquid does not occur, and this matched instantaneous change in the pressures is possible. If we consider also its own volume of cracks, then will appear also another independent combination of pressures p', which determines a change in the effective volume of cracks. In this case, both pressures $p_1$ and $p_2$ will prove to be continuous with $t = 0$, and will have to assign their initial values separately.

Another special feature/peculiarity of system (VII.1.11) entails the fact that in it is excluded after smallness the fluid flow directly on porous blocks.

Page 192.

Therefore the equalization of a difference in the pore pressures $p_2$ between two adjacent points of the medium can occur only by means of the exchange of the liquid between blocks and cracks and the displacement of liquid over cracks. As a result of this in the cracked-porous medium, described by equations (VII.1.11), the jumps

of pore pressure do not disappear instantly (as this is, for example, during elastic mode/conditions), but they attenuate in time exponentially. In order to be convinced of this, we will set up condition on the jumps which must be fulfilled for the solutions of system (VII.1.11).

Let us examine the isolated/insulated discontinuity surface of $\Sigma$. During the conclusion/derivation of conditions on its jumps it is possible to consider plane and to accept for plane x = 0.

Let us integrate the second equation (VII.1.11) for x within limits from $-\varepsilon$ to $\varepsilon$. On the strength of the limitedness of $p_2$, $p_1$, $\frac{\partial^2 p_1}{\partial x^2}$ and $\frac{\partial^2 p_1}{\partial y^2}$ with cf $\varepsilon \to 0$ we have

$$\frac{\partial p_1}{\partial x}\Big|_{-\varepsilon}^{\varepsilon} = \int_{-\varepsilon}^{\varepsilon} \left[ \frac{A}{\varkappa}(p_2 - p_1) - \frac{\partial^2 p_1}{\partial x^2} - \frac{\partial^2 p_1}{\partial y^2} \right] dx \to 0.$$

. Thus, the derivative $dp_1/dx$, and together with it and pressure itself in cracks $p_1$ are continuous on the surface of $\Sigma$.

We will write now the first equation of system (VII.1.11) for points in front of discontinuity surface ($x = +0$) and for points after this surface ($x = -0$), designating appropriate values by the marks of $+$ and $-$ and let us deduct the obtained equations from each other. We have

$$\frac{\partial(p_2^+ - p_2^-)}{\partial t} - \beta\frac{\partial(p_1^+ - p_1^-)}{\partial t} + A[(p_2^+ - p_1^+) - (p_2^- - p_1^-)] = 0.$$

On demonstrated $[p_1] = p_1^+ - p_1^- = 0,$ so that for the pressure shock of $[p_2] = p_2^+ - p_2^-$ we have

$$\frac{\partial[p_2]}{\partial t} + A[p_2] = 0. \qquad \text{(VII.1.14)}$$

Thus, the jumps of the pore pressure $p_2$ must satisfy an equation (VII.1.14), or after integration

$$[p_2] = [p_2]_0 e^{-At}. \qquad \text{(VII.1.15)}$$

Here by the $[p_2]_0$ is designated the initial jump at torque/moment t = 0.

We allow now, that near the surface of $\Sigma'$ (by being or not being discontinuity surface of pressure $p_2$) derived $\partial p_1/\partial x$ is continuous. then the first equation (VII.1.11) possible outside the surface of $\Sigma'$ (taken for plane x = 0) to differentiate on x, after obtaining in this case

$$\frac{\partial}{\partial t}\left(\frac{\partial p_2}{\partial x}\right)+A\frac{\partial p_2}{\partial x}=0. \qquad \text{(VII.1.16)}$$

. Page 193.

Applying to this equation the same reasonings, as above, and by utilizing continuity of derived $\partial p_1/\partial x$ on the surface of $\Sigma'$, we will obtain

$$\frac{\partial}{\partial t}\left[\frac{\partial p_2}{\partial x}\right]+A\left[\frac{\partial p_2}{\partial x}\right]=0,$$

$$\left[\frac{\partial p_2}{\partial x}\right]=\left[\frac{\partial p_2}{\partial x}\right]_0 e^{-At}. \qquad \text{(VII.1.17)}$$

4. The noted in the preceding/previous point/item special feature/peculiarities of the solutions of equation (VII.1.12) and of system (VII.1.11) give rise to the appropriate special feature/peculiarities in the setting of boundary and initial conditions, by which they must satisfy these solutions.

First of all, as already it was said earlier, it cannot be required, in order to with tendency t to zero both pressures (in pores and cracks) they accepted predetermined value of $p_2$ (0, x, y,

z); $p_2$ (0, x, y, z). Necessary conditions must be only the continuity of reduced pressure

$$p = p_2 - \beta p_1. \qquad \text{(VII.1.18)}$$

a pressure in cracks $p_1$ must then be determined from equation (VII.1.13). Thus, the initial condition will take the form:

$$p(0, x, y, z) = p_2(0, x, y, z) - \beta p_1(0, x, y, z) = f(x, y, z). \qquad \text{(VII.1.19)}$$

In turn, with tendency to boundary of the region only pressure in cracks $p_1$ must be continuously together with its derivatives. As usual, we will examine the conditions of three types: when on boundary are assigned values of the pressure of liquid and fluid flow or their combination, i.e., conditions of the type:

$$p_1(t, S) = \varphi(S), \quad \frac{\partial p_1(t, S)}{\partial n} = \varphi(S)$$

or

$$p_1(t, S) + h\frac{\partial p_1(t, S)}{\partial n} = \chi(S). \qquad \text{(VII.1.20)}$$

. Here S designates the point of bounding surface, and n - the direction of standard to it.

Description of the cracked-porous medium as "the dual" porous medium or the systems of the inserted into each other porous media, capable of being interchanged with liquid, is given in the works of G. I. Barenblatt, S. P. Zheltova and I. N. Kochinoy [17, 18]; there are examined some examples. The The statement of boundary-value problems for equation (VII.1.12) is refined in work [14].

Another approach to the description of the cracked-porous media, connected with the examination of the routinely arranged/located cracks, belongs to Ye. S. Rome with co-authors (for example, see [97]).

Page 194.

§2. Basic problems of the unsteady filtration of homogeneous liquid in cracked and laminar layers.

1. The problems of the unsteady filtration in the cracked-porous medium the more complex appropriate problems of the theory of elastic mode/conditions, because their describing equations (VII.1.12) have not the second, and third order and they do not allow/assume self-similar solutions, since contain characteristic time the aaaaaaaaa. Let us examine here two simplest problems, which are of greatest practical interest, the problem of inflow to drainage gallery and of the launching/starting of hole.

Let us assume that initially the pressure in all cracked-porous layer, which occupies half-space x :ml 0, constantly is equal to zero, but at torque/moment t = 0, boundary of layer x = 0 is communicated with the region of the constant pressure $p = P_1$. The problem of the determination of pressure in the porous blocks $p_2$ is reduced to the solution to equation (it is accepted $\beta$ = by 0)

$$\frac{\partial p_2}{\partial x} - \eta \frac{\partial^3 p_2}{\partial x^2 \partial t} = \varkappa \frac{\partial^2 p_2}{\partial x^2} \qquad \text{(VII.2.1)}$$

under the supplementary conditions:

$$p_2(0, x) = 0 \qquad (0 \leqslant x < \infty);$$
$$p_2(t, +0) = P_1(1 - e^{-\varkappa t/\eta}). \qquad \text{(VII.2.2)}$$

The form of boundary condition with $x = 0$ is connected with the law of fading the breakings of the distribution of pore pressure, minutely examined in the preceding/previous paragraph.

Converting equation (VII.2.1) according to Laplace and taking into account the initial condition, we obtain

$$\frac{d^2\bar{p}_2}{dx^2} - \frac{\sigma\bar{p}_2}{\sigma\eta + \varkappa} = 0, \qquad \text{(VII.2.3)}$$

where

$$\bar{p}_2 = \bar{p}_2(\sigma, \varkappa) = \int_0^\infty e^{-\sigma t} p_2(t, x)\, dt. \qquad \text{(VII.2.4)}$$

Conversion of boundary conditions (VII.2.2) gives

$$\bar{p}_2(\sigma, +0) = \frac{\varkappa P_1}{\sigma(\sigma\eta + \varkappa)}; \quad \bar{p}_2(\sigma, \infty) = 0. \qquad (VII.2.5)$$

the satisfying these conditions solution to equation takes the form:

$$\bar{p}_2(\sigma, x) = \frac{\varkappa P_1}{\sigma(\sigma\eta + \varkappa)} \exp\left[-x\sqrt{\frac{\sigma}{\sigma\eta + \varkappa}}\right]. \qquad (VII.2.6)$$

Utilizing an inversion formula, we obtain

$$p_2(t, x) = \frac{\varkappa P_1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{e^{\sigma t}}{\sigma(\sigma\eta + \varkappa)} e^{-\sqrt{\frac{\sigma}{\sigma\eta+\varkappa}}x} d\sigma. \qquad (VII.2.7)$$

Page 195.

During the calculation of integral (VII.2.7) it is convenient to bring together it to integral in terms of the duct, which covers the negative part of the real axis. In this case, as it is easy to verify that integrals in terms of the section of $(-\infty, -\varkappa/\eta)$ average out and it remains only integral in terms of the duct, which covers the cut of $(-\varkappa/\eta, 0)$. Then

$$p_2(t, x) = P_1\left(1 - \frac{1}{\pi}\int_0^1 e^{-\sigma t\varkappa/\eta} \sin\left(\sqrt{\frac{\sigma}{1-\sigma}}\,\frac{x}{\sqrt{\eta}}\right)\frac{d\sigma}{\sigma(1-\sigma)}\right) \quad \text{(VII.2.8)}$$

(the term outside the integral here is obtained as a result of integration for a small duct, which covers point $\sigma = 0$).

Let us place, $\sigma/(1-\sigma)=\nu^2\eta$. We have

$$p_2(t,\,x)=P_1-\frac{2P_1}{\pi}\int\limits_0^\infty\frac{\sin\nu x}{\nu}\exp\left(-\frac{\nu^2\varkappa t}{1+\nu^2\eta}\right)d\nu=$$

$$=P_1-\frac{2P_1}{\pi}\int\limits_0^\infty\frac{\sin u}{u}\exp\left(-\frac{u^2}{4\xi^2+\eta u^2/\varkappa t}\right)du;\ \xi=\frac{x}{2\sqrt{\varkappa t}}.\quad\text{(VII.2.9)}$$

. Hence with $\eta=0$ is obtained pressure distribution during the launching/starting of gallery in the homogeneous porous medium

$$p(t, x) = P_1 - \frac{2P_1}{\pi}\int_0^{\infty} \frac{\sin u}{u} \exp\left(-\frac{u^2}{4\xi^2}\right) du = P_1 (1 - \operatorname{erf} \xi). \qquad \text{(VII.2.10)}$$

In order that instead of (VII.2.9) it was possible to use usual relationship/ratio (VII.2.10), necessary the execution of inequality

$$\eta/\varkappa t \ll 1. \qquad \text{(VII.2.11)}$$

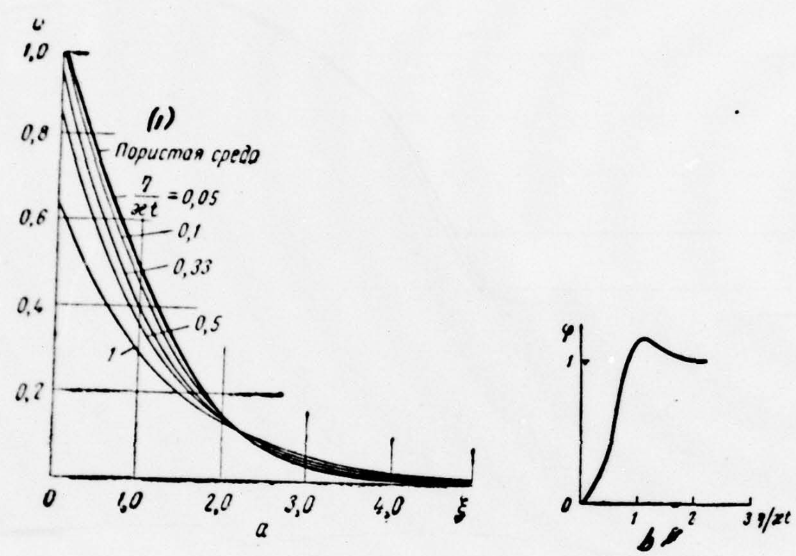
Actually, in this case the term of $\eta u^2/\varkappa t$ it is compared from the $\xi^2$ only with of the $u^2 \gg \xi^2$, when exponential factor in (VII.2.9) is already negligible. If we, on the contrary, replace inequality (VII.2.11) by reverse/inverse, then the index of the exponential is low with all u, and equation (VII.2.9) gives

$$p_2(x, t) \approx P_1 - \frac{2P_1}{\pi} \int_0^{\infty} \frac{\sin u}{u}\, du = 0. \qquad \text{(VII.2.12)}$$

Thus, with the times, greater in comparison with characteristic time of aaaaaaa pressure in the blocks of cracked-porous layer is changed just as in usual porous layer. With times small in comparison with aaaaaaa the pressure in blocks is not changed at all. Appearing thus delay is characteristic for the cracked-porous medium.

Fig. VII.2.

Key: (1). Porous medium.



Let us compute now the law of a change in the fluid flow through the boundary of layer x = 0. Since the flow is proportional to derivative of pressure in cracks $p_1$, it is necessary first of all to calculate this pressure. For this, it is convenient to use the first equation of the basic system (VII.1.11). Set/assuming $\beta = 0$, we obtain

$$p_1 = p_2 + \frac{\eta}{\varkappa} \frac{\partial p_2}{\partial t}. \qquad \text{(VII.2.13)}$$

With any $t \neq 0$ differentiation in (VII.2.9) can be produced under the integral. We have

$$p_1 = p_2 + \frac{\eta}{\varkappa}\frac{\partial p_2}{\partial t} = P_1 - \frac{2P_1}{\pi}\int_0^\infty \frac{\sin \nu x}{\nu(1+\nu^2\eta)} \times \exp\left(-\frac{\nu^2 \varkappa t}{1+\eta\nu^2}\right) d\nu. \quad \text{(VII.2.14)}$$

From (VII.2.14), in particular, it follows $p_1(0, t) = P_1$, as this but had to be in accordance with what has been said earlier. In order to calculate fluid flow through boundary of $x = 0$, it is necessary to differentiate expression (VII.2.14) on x with $x = 0$. Differentiating under the integral, we have

$$q=\frac{\partial p}{\partial x}\Big|_{x=0}=-\frac{2P_1}{\pi\sqrt{\varkappa t}}\int\limits_0^\infty \exp\left[-\frac{u^2}{1+u^2\eta/\varkappa t}\right]\frac{du}{1+u^2\eta/\varkappa t}. \quad \text{(VII.2.15)}$$

Under conditions of the elastic mode/conditions of $q=q_0=-P_1/\sqrt{\pi\varkappa t}.$ Thus,

$$\frac{q}{q_0}=\sqrt{\frac{\pi\varkappa t}{\eta}}\exp\left(-\frac{\eta}{2\varkappa t}\right)I_0\left(\frac{\eta}{2\varkappa t}\right)=\varphi\left(\frac{\eta}{\varkappa t}\right). \quad \text{(VII.2.16)}$$

Figure VII.2a shows pressure distributions in pores for the different values of the parameter of $\eta/\varkappa t$ and in Fig. VII.2b - the function of $\varphi(\eta/\varkappa t)$. As one would expect, with $t\rightarrow\infty$ $(\eta\rightarrow 0)$ all the soluticns approach the appropriate solutions for the porous medium.

Page 197.

2. Unsteady motion near the hole, which works with constant flow rate/consumption. Let us examine now axisymmetric problem, by assuming that into layer, which is located at constant pressure $p_0$ = by 0, begins the pumping in of liquid with constant flow rate/consumption of Q through the hole of negligibly small radius.

In cylindrical coordinates the problem in question is reduced to the solution to equation

$$\frac{\partial p_1}{\partial t} - \eta \frac{\partial}{\partial t}\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial p_1}{\partial r}\right) = \varkappa \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial p_1}{\partial r}\right) \qquad \text{(VII.2.17)}$$

under the conditions

$$p_1(0, r) = 0;\; p_1(t, \infty) = 0;\; \left(r \frac{\partial p_1}{\partial r}\right)_{r=0} = -\frac{\mu Q}{2\pi k_1 h} = -p_*. \qquad \text{(VII.2.18)}$$

Problem (VII.2.17-18) is formulated for a pressure in cracks $p_1$; with its wish it is possible to formulate for a pressure in the porous assemblies $p_2$. Then boundary condition with r = 0 signs the form:

$$r\left(\frac{\partial p_2}{\partial r}\right)_{r=0}+\frac{\eta}{\varkappa}\frac{\partial}{\partial t}\left(r\frac{\partial p_2}{\partial r}\right)_{r=0}=-p_*, \qquad \text{(VII.2.19)}$$

and remaining conditions and the fundamental equation they will remain without change.

Transfer/converting in relationship/ratios (VII.2.17-18) to Laplace transforms, we have

$$\frac{1}{r}\frac{d}{dr}r\frac{dp_1}{dr}-\frac{\sigma}{\varkappa+\eta\sigma}\bar{p}_1=0;\ \left(r\frac{\partial\bar{p}_1}{\partial r}\right)_{r=0}=-\frac{p_*}{\sigma};\ \bar{p}_1(\infty)=0. \qquad \text{(VII.2.20)}$$

These conditions are satisfied by solution

$$\bar{p}_1 = \frac{p_*}{\sigma} K_0\left(\sqrt{\frac{\sigma}{\varkappa + \sigma\eta}}\, r\right). \qquad \text{(VII.2.21)}$$

so that by inversion formula

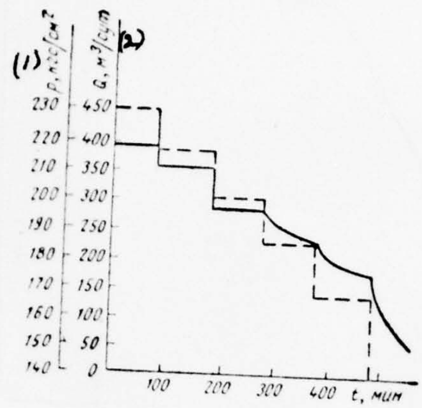
$$p_1(t, r) = \frac{p_*}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{e^{\sigma t}}{\sigma} K_0\left(\sqrt{\frac{\sigma}{\varkappa + \sigma\eta}}\, r\right) d\sigma. \qquad \text{(VII.2.22)}$$

This integral can be brought to integral in terms of real variable in the same way as this was made in the preceding/previous point/item. We, however, analyze only the asymptotic behavior of the obtained solution in the low values of the parameter of $\rho = r/2\sqrt{\varkappa t}$. Let us present expression (VI.2.20) in the form:

$$p_1(t, r) = \frac{p_*}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{e^{\zeta}}{\zeta} K_0\left(r \sqrt{\frac{\zeta}{1 + \zeta\eta/\varkappa t}}\right) d\zeta \qquad \text{(VII.2.23)}$$

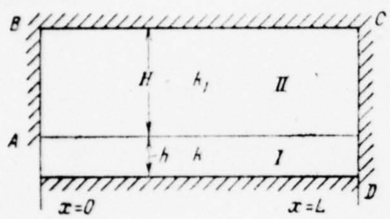
We will count $\rho \ll 1$.

Fig. VII.3.




Key: (1). kgf/cm². (2). m³/day.

Fig. VII.4.



With $\eta/\varkappa t \ll 1$ the expression in question transfer/converts to the known expression of the theory of elastic mode/conditions (comp. §2 chapter III). But if $\eta/\varkappa t \gg 1$, then the expression, which stands under the sign of the function of Macdonald, is evenly small, so that for it it is possible to use the approximate representation

$$K_0(x) = -(C + \ln x/2) + o(1).$$

As a result we obtain

$$p_1(t, r) = -p_*\left(C + \ln \frac{r}{2\sqrt{\eta}}\right)$$
$$(r/\sqrt{\varkappa t} \ll 1, \ \varkappa t/\eta \ll 1). \qquad \text{(VII.2.24)}$$

the sense of relationship/ratio (VII.2.24) is simple: it means that if the proper time of the cracked-porous medium of $\eta/\varkappa$ not

is toc small, there is an intermediate quasi-stationary mode/conditions, when the liquid, which enters from hole, is absorbed by the nearest to it assemblies. If and only it pressure in assemblies in the vicinity of hole will be equalized with pressure in cracks (i.e. after time of $\sim\eta/\varkappa$), begins to pronounce the exchange of liquid with the more distant sections of layer [1].

FOOTNOTE [1]. Let us note one additional fact. the relationship/ratio (VII.2.24) shows that there is certain time interval of

$r^2/\varkappa \ll t \ll \eta/\varkappa$, in extent/elongation of which pressure in hole it is not changed. If by time of $r^2/\varkappa$ it is possible to disregard (usually these are the hundredth fractions of second and less), then of (VII.2.24) it follows that during an abrupt change of the output of hole the pressure in it changes with jump, and then retains constant value in the duration of $\sim\eta/\varkappa$. This behavior of pressure really/actually is observed in practice. In Fig. VII.3, borrowed from work [80], is shown a stepped variation in the pressure (several, true, distorted by the effect of incidental factors). ENDFOOTNOTE.

Close in the character of problem appear during the study of filtration in laminar layers. For example if motion occurs in two

lying/horizontal above each other layers, isolated by the slightly permeable cross connection, then pressure in each of the layers follows to the equation of elastic mode/conditions with the intensity of the return flow between layers in right side; this intensity in the majority of cases can be counted a proportional pressure difference at the corresponding points of layers.

Page 199.

The resemblance of the appearing problem to the problem of filtration in the "dual" porous medium is obvious.

From entire diversity of the problems of this cycle, we will examine here only one - the problem of depletion of the layer, which borders to the layer of large power/thickness, but small permeability. This problem is of large interest in connection with the estimation of the supplies of oil and gas of some deposits.

Let us assume that the region of filtration takes the form, given in Fig. VII.4. Let us assume that layers I and II are

accumulated by the rock/species of identical porosity, but essentially different permeability, so that kh >> $k_1H$, although H >> h.

We will examine depletion of system, assuming that at first it was found $P_0$, and from torque/moment t = 0, begins the selection of the liquid through the lower layer in section x = 0, whereupon pressure on the entire line x = 0 equally, and the selection of liquid Q is retained constant. system is considered locked, i.e., boundaries AB, BC and CD are impenetrable. In this case, the problem is reduced to the sclution of the totality of equations

$$\frac{\partial p}{\partial t} = \varkappa\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \qquad (-h \leqslant y \leqslant 0); \quad (0 \leqslant x \leqslant L);$$
$$\frac{\partial p}{\partial t} = \varkappa_1\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \qquad (0 \leqslant y \leqslant H). \qquad \text{(VII.2.25)}$$

. Under the conditions

$$k_1 \frac{\partial p}{\partial y}\bigg|_{y=+0} = k \frac{\partial p}{\partial y}\bigg|_{y=-0}; \; p|_{y=+0} = p|_{y=-0} \qquad \text{(VII.2.26)}$$

$$\frac{\partial}{\partial y} p(0, y, t) = 0; \; k \int_{-h}^{0} \frac{\partial p}{\partial x}\bigg|_{x=0} dy = Q; \; \frac{\partial p}{\partial x}\bigg|_{x=0} = 0 \quad (y>0). \qquad \text{(VII.2.27)}$$

Under the made assumptions (fine/thin lower and slightly permeated upper layers) to the statement of problem it is possible to simplify.

Let us note first of all, that on the strength of the equality of the boundary values of pressure in both layers with $y = 0$ derivatives in terms of $x$ of pressure in these layers - one order, and consequently, the rate of filtration in the direction of X-axis in the upper layer is negligible (according to condition $k_1 H << kh$). At the same time the rates of filtration in the direction of $y$ axis coincide with $y = 0$, which can be only if pressure change in the direction of $y$ axis in the upper layer occurs faster than in lower. Hence follows

$$\partial^2 p/\partial y^2 \gg \partial^2 p/\partial x^2.$$

Thus, the second equation (VII.2.25) can be written in the form:

$$\frac{\partial p}{\partial t} = \varkappa_1 \frac{\partial^2 p}{\partial y^2} \qquad (y>0).$$

Page 200. The first equation, which relates to lower layer, can be averaged according to power/thickness. Set/assuming $P = \frac{1}{h}\int\limits_{-h}^{0} p dy,$ we have

$$\frac{\partial P}{\partial t} = \varkappa \frac{\partial^2 P}{\partial x^2} + \frac{\varkappa}{h}\left(\frac{\partial p}{\partial y}\right)_{y=-0} = \varkappa \frac{\partial^2 P}{\partial x^2} + \frac{\varkappa k_1}{hk}\left(\frac{\partial p}{\partial y}\right)_{y=+0}$$

(is here used the boundary condition of

$$k_1 \frac{\partial p}{\partial y}\Big|_{y=+0} = k\frac{\partial p}{\partial y}\Big|_{y=-0}).$$

Finally, let us replace the condition of $p|_{y=+0} = p|_{y=-0}$ with the condition of $p|_{y=+0} = P$. The completed in this case error is small with the small thickness h of lower layer.

Thus, appears the following simplified task:

$$\frac{\partial p}{\partial t} = \varkappa_1 \frac{\partial^2 p}{\partial y^2} \qquad [p = p(x, y, t);\ y > 0];$$

$$\frac{\partial P}{\partial t} = \varkappa \frac{\partial^2 P}{\partial x^2} + \frac{\varkappa k_1}{hk}\left(\frac{\partial p}{\partial y}\right)_{y=0} \qquad [P = P(x, t)]; \qquad \text{(VII.2.28)}$$

$$p(x, y, 0) = P(x, 0) = P_0; \quad p(x, 0, t) = P(x, t); \qquad \text{(VII.2.29)}$$

$$\frac{\partial p}{\partial y}\Big|_{y=H} = 0; \ \frac{kh}{\mu}\frac{\partial P}{\partial x}\Big|_{x=0} = Q = \text{const}; \ \frac{\partial P}{\partial x}\Big|_{x=L} = 0. \qquad \text{(VII.2.30)}$$

Solution of this problem easily to obtain by operating method.

Without giving it completely, let us extract formula for image $P_0$ from pressure on gallery $P_0 = P\ (0,\ t)$.

We have

$$\bar{p}_0 = \frac{p_0}{\sigma} - \frac{q\sqrt{\varkappa}}{\sigma}\,\frac{\operatorname{cth}\left[L\sqrt{\frac{\sigma}{\varkappa} + \frac{k_1}{kh}\sqrt{\frac{\sigma}{\varkappa_1}}\operatorname{th}\left(H\sqrt{\frac{\sigma}{\varkappa_1}}\right)}\right]}{\sqrt{\sigma + \frac{k_1\varkappa}{kh}\sqrt{\frac{\sigma}{\varkappa_1}}\operatorname{th}\left(H\sqrt{\frac{\sigma}{\varkappa_1}}\right)}}. \quad \text{(VII.2.31)}$$

It is hence easy to obtain several simple expressions, which correspond different times from the torque/moment of the launching/starting of gallery.

Let first of all time t be is so small that the perturbations that arose on gallery, did not achieve the impenetrable boundaries of system;

$$t \ll L^2/\varkappa \ll H^2/\varkappa_1. \quad \text{(VII.2.32)}$$

In this case, in equation (VII.2.32) it is possible to restrict ourselves to the asymptotic behavior of

$\sigma \gg \varkappa L^{-2} \gg \varkappa_1 H^{-2}$. For such values $\sigma$ hyperbolic tangent and cotangent in (VII.2.31) it is possible to replace them with extreme values with $\sigma \to \infty$, equal to unity, whence

$$p_0 \approx \frac{P_0}{\sigma} - \frac{q\sqrt{\varkappa}}{\sigma\sqrt{\sigma + \frac{k_1\varkappa}{kh}\sqrt{\frac{\sigma}{\varkappa_1}}}}.$$

. Page 201.

Taking into account still that in this case of $\frac{k_1\varkappa}{kh\sqrt{\sigma\varkappa_1}} \ll 1$, we have

$$\bar{p}_0 \approx -\frac{q\sqrt{\varkappa}}{\sigma\sqrt{\sigma}}\left(1-\frac{k_1\varkappa}{2kh\sqrt{\varkappa_1\sigma}}+\ldots\right)+\frac{P_0}{\sigma}, \qquad \text{(VII.2.33)}$$

consequently,

$$p_0(t) \approx P_0 - 2\sqrt{\frac{\varkappa t}{\pi}}\,q + \frac{k_1\varkappa}{2kh}\sqrt{\frac{\varkappa}{\varkappa_1}}\,tq + \ldots \qquad \text{(VII.2.34)}$$

Thus, during the first stage of motion the effect of the upper slightly permeable layer pronounces only in the addition of low terms, order of $\sqrt{t}$, in comparison with main.

Let us examine now the intermediate range of times, which answers the expansion of the formula (VII.2.31) with of $\varkappa L^{-2} \gg \sigma \gg \varkappa_1 H^{-2}$. In this case again it is possible to place $thH\sqrt{\frac{\sigma}{\varkappa_1}} = 1$, a $\mathrm{cth}\left[L\sqrt{\frac{\sigma}{\varkappa_1}}+\ldots\right]$, by utilizing the second inequality for $\sigma$, it is possible to present in expansion at the low

values of argument. We have

$$\bar{p} = \frac{P_0}{\sigma} - \frac{q\varkappa}{L\sigma\left(\sigma + \frac{k_1\varkappa}{kh}\sqrt{\frac{\sigma}{\varkappa_1}}\right)} + \dots \qquad \text{(VII.2.35)}$$

Hence, utilizing table of Laplace's images, we obtain

$$p_0(t) = P_0 - \frac{q\varkappa}{L}\left[\frac{k^2h^2\varkappa_1}{\varkappa^2k_1^2}\exp\left(\frac{k_1^2\varkappa^2}{k^2h^2\varkappa_1}t\right)\times\right.$$
$$\left.\times\,\mathrm{erfc}\left(\frac{\varkappa k_1\sqrt{t}}{kh\sqrt{\varkappa_1}}\right) + \frac{2\sqrt{t\varkappa_1}}{\sqrt{\pi}}\,\frac{kh}{\varkappa k_1} - \frac{k^2h^2\varkappa_1}{\varkappa^2k_1^2}\right]. \qquad \text{(VII.2.36)}$$

The last/latter expression can be simplified, if the value of

$\frac{k_1\varkappa}{kh}\sqrt{t}$ is low or great in comparison with unity. In the first case, decompose/expanding the first term in brackets according to the degrees of argument, we will obtain

$$p_0(t) \approx P_0 - \frac{q\varkappa t}{L} + \ldots, \qquad \text{(VII.2.37)}$$

that it coincides with solution for the case of impenetrable upper layer. But if the $k_1\varkappa\sqrt{t}/kh \gg 1$, then for the sake of simplicity in the first term it is possible to utilize asymptotic behavior erfc x at great significance of argument

$$\operatorname{erfc} x \approx \frac{1}{\sqrt{\pi}x} e^{-x^2}.$$

We have

$$p_0(t) = -\frac{\varkappa q}{L}\left(\frac{kh}{\varkappa k_1} 2\sqrt{\frac{t\varkappa_1}{\pi}} - \frac{k^2h^2\varkappa_1}{\varkappa^2 k_1^2} + o(1)\right) + P_0. \qquad \text{(VII.2.38)}$$

. Page 202.

Thus, the pressure change in this case is determined already in essence by inflow from the upper layer. Finally, with even longer times of $t \gg H^2/\varkappa_1$, begins the second phase of filtration in the upper layer (depletion of the upper layer). Decompose/expanding hyperbolic tangent and cotangent at the low values of argument, we obtain

$$\bar{p}_0 = \frac{P_0}{\sigma} - \frac{q\varkappa}{\sigma^2 L\left(1 + \frac{\varkappa k_1 H}{\varkappa_1 kh}\right)} = \frac{P_0}{\sigma} - \frac{q\varkappa h}{\sigma^2 L\left(h + H\frac{Km_1}{K_1 m}\right)}, \quad \text{(VII.2.39)}$$

whence

$$p_0(t) \approx P_0 - \frac{q\varkappa ht}{L\left(h + H\frac{Km_1}{K_1 m}\right)} \ldots = P_0 - \frac{Qt}{L}\frac{1}{\left(\frac{hm}{K} + \frac{Hm_1}{K_1}\right)} + \ldots \quad \text{(VII.2.40)}$$

Thus, for the two-layered layer of the form in question distinctly isolated two cycle of motion in work for depletion. In the extent/elongation of the first period, occurs the depletion of the first layer, and motion in slightly-permeable upper layer is insignificant, at the second stage the lower layer is practically completely exhausted, and occurs depletion of the upper layer.

If we according to data on a pressure drop with selection during the first stage calculate the supplies of liquid or gas in layer, then calculation will give only the supplies, included in lower layer ($V_0$ = mhbL), that it is considerably less than the true supplies of V = (mh + $m_1$H) bL. This fact turns out to be essential for a series of deposits; specifically, that it is matter on the greatest shebelinka deposit of gas. As showed M. A. Bernstein [28a], the initial supplies of shebelinka deposit turned out to be those which were understated, since were not taken into account the supplies of gas in the slightly

permeable anhydrite. The permeability of these rock/species is so small that the drilled in them holes do not have industrial value however return flow of the anhydrite into the arranged/located below well penetrated layer it turned out to be very essential.

§3. Two-phase nonstationary filtration and the displacement of the nonmiscible liquids in the media with dual porosity.

During consistent motion in the heterogeneous porous medium of two nonmiscible liquids, appear the supplementary factors, which cause the exchange of the liquid between the sections of different permeability. First of all because of a difference of the saturation in the different regions of the porous medium, is changed filtration resistance, which causes the redistribution of pressure and the return flows of the liquid between the highly and slightly-permeable sections. Another reason, which causes the exchange of the liquid between the high-permeability medium and slightly-permeable connection/inclusions during the displacement of the nonmiscible liquids, consists in action of capillary forces.

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THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS (TEORIY--ETC(U)
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3 OF 5
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Page 203.

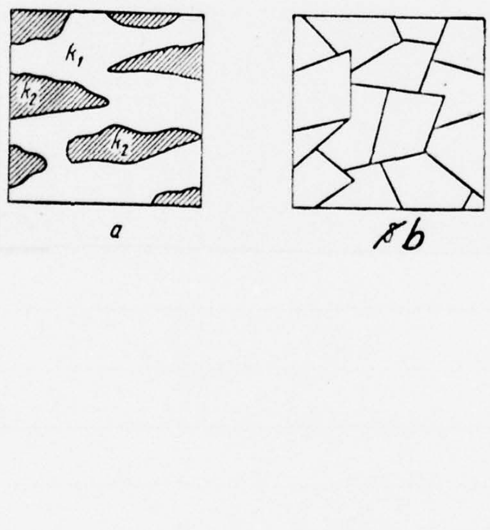


Fig. VII.5.

The at first displacing liquid (water) rapidly bursts open on the high-permeability medium, and slightly-permeable connection/inclusions prove to be surrounded water. Therefore the containing in them oil can be extracted only by means of countercurrent capillary impregnation if the displacing phase it is more wetting.

For the description of the displacement of the nonmiscible liquids from the media with dual porosity, can be applied common/general/total the approach, presented in the preceding/previous paragraphs. We will examine the displacement of the nonmiscible liquids in the medium which consists of region with permeability $k_1$, which has slightly-permeable connection/inclusions with permeability $k_2 << k_1$. the skeletal diagram of this medium is depicted on Fig. VII.5a. Since $k_2 << k_1$, filtration in slightly-permeable sections because of common/general/total pressure gradient in layer we will disregard. Special cases of the diagram in question are the layer, which consists of two layers of different permeability, and the cracked-porous layer (Fig. VII.5b).

The processes of the redistribution of pressure because of the compressibility of the liquid and porous medium, especially in the

cracked-porous medium, occur considerably faster than the redistribution of liquid, caused by capillary forces, since the piezoconductivity of the porous media, as already mentioned in chapter II, it is of the order $10^4$ cm²/s, but the parameter of

$$a^2 = \frac{\alpha}{\mu}\sqrt{\frac{k}{m}},$$ which determines the rate of capillary processes, almost never exceeds 1 cm²/s. Therefore in the majority of cases it is possible to be restricted to the investigation of the displacement of incompressible liquids in the incompressible porous medium. one case of the displacement of compressible liquids in the medium with dual porosity will be examined at the end of the present paragraph.

As for the description of the unsteady filtration of homogeneous liquid in the medium with dual porosity, let us introduce at each point of continuous medium on two values of each characteristic of the driving liquid - one for slightly-permeable medium, another - for high-permeability. Thus, we will utilize two rates of filtration each of the phases $\vec{U}_i^{(1)}$ and an $\vec{U}_i^{(2)}$, two saturation of $s^{(1)}$ and $s^{(2)}$ and two pressure of $p^{(1)}$ and $p^{(2)}$ (superscripts 1 and 2 are related respectively to high-permeability and slightly-permeable to the media, lower 1 and 2 - to that which displace and that which is displaced to liquids). Each of the indicated characteristics is obtained by means of averaging by the volume, which covers the large number of assemblies or

slightly-permeable incorporations. By the laminar medium the averaging is conducted according to the power/thickness of each layer.

Page 204.

In accordance with condition $k_1 >> k_2$, we will examine the only case when entire filtration occurs in the high-permeability medium, i.e., $\vec{U}_i^{(2)}$ are considered equal to zero. The generalized law of darcys for motion in the high-permeability medium can be written in the form:

$$\vec{U}_1^{(1)} = -\frac{k_1}{\mu_1}\bar{f}_1^{(1)} \operatorname{grad} p^{(1)};$$
$$\vec{U}_2^{(1)} = -\frac{k_1}{\mu_2}\bar{f}_2^{(1)} \operatorname{grad} p^{(1)}. \qquad \text{(VII.3.1)}$$

In formulas (VII.3.1) the pressure is accepted identical in both phases, i.e., by the effect of capillary forces directly on

filtration it is disregarded. Their action is considered, further, only through the rate of the exchange of the liquid between the sections of different permeability. This simplification correctly, if the zone where substantially is changed saturation, includes the large number of assemblies (by the laminar medium this zone must considerably exceed the power/thickness of each of the layers). Then the gradient of saturation in the "longitudinal" direction in the high-permeability medium is small, and the effect of capillary forces on filtration can be disregarded. This condition is satisfied feast the sufficiently low value of the parameter of the $a_1^2/Uh$, which determines the ratio of the rate of capillary impregnation to rate of filtration.

The relative permeability of $\bar{f}_i^{(1)}$, entering the formulas (VII.3.1), can differ significantly from the relative permeability, which enter the equations of the generalized law of darcys for the homogeneous porous media (with identical saturation). The latter are related to the quasi-static distributions of saturation which in macroscale are uniform, and in microscale they are determined by the action of surface forces (see Chapter VI). In inhomogeneous medium due to the irregular form of slightly-permeable incorporations and under the effect of other reasons the distribution of saturation during unsteady flow noticeably differs from the quasi-static. In the

high-permeability medium can be formed the regions, in which the saturation sharply differs from average, the "languages" of water, the pillars of oil. It is natural that in this case the preferred location of particles of the more wetting phase in pin-head blisters is disrupted, and the form of the averaged curves of relative permeability approaches rectilinear (rectilinear relative permeability correspond completely to the random distribution of phases in pores).

The form of the curves of relative permeability in inhomogeneous medium depends on the character of heterogeneity. Comparatively uniform in macroscale will be phase distribution according to the power/thickness of the high-permeability layer in laminar layer, if this power/thickness is less than the power/thickness slightly-permeable layer.

Page 205.

In this case the curves of relative permeability retain usual form. On the contrary, in cracked-porous medium relative permeability during the motion of phases in cracks can be in the majority of cases

considered the linear functions of saturation, since phase distribution in cracks does not depend on capillary forces and each phase moves freely (experimentally this fact checked by Ye. S. Rome [97]). The relative permeability in the high-permeability medium are also the functions of the dimensionless parameters of form $a^2/Uh$, where U is a rate of displacement, h - the significant dimension of inclusion. however, this dependence thus far is not investigated. Further let us assume that the averaged relative permeability depend only on the average saturation of the corresponding medium.

the equations of continuity can be extracted for each phase in each of the constituting media analogous with that, as this was made in §1 for a uniform liquid, i.e., by assuming that at each point occurs the exchange of the liquid between the appropriate media with an intensity of $q_1$.

We have for flow in the high-permeability medium

$$m_1(1-n)\frac{\partial s^{(1)}}{\partial t}+\operatorname{div}[1-n)\vec{U}_1^{(1)}]+q_1=0;$$
$$-m_1(1-n)\frac{\partial s^{(1)}}{\partial t}+\operatorname{div}[(1-n)\vec{U}_2^{(1)}]+q_2=0. \quad \text{(VII.3.2)}$$

. Here $q_1$ and $q_2$ are intensities of the return flow of each phase from the more penetrated medium less penetrated; n - the bulk density of inclusions, which depends on space coordinates. If we assume in accordance with what has been said it is above that in slightly-permeable switchings on of rate of filtration the $\vec{U}_i^{(2)}$ are equal to zero, then the equations of continuity in slightly-permeable medium take the form:

$$m_2 n \frac{\partial s^{(2)}}{\partial t} - q = 0;$$
$$q_1 = -q_2 = q, \qquad \text{(VII.3.3)}$$

i.e. the return flows of each of the phases are equal in magnitude and are opposite in the direction.

For the closing/shorting of the obtained system of equations, it is necessary to find communication/connection of the intensity of return flows with other variables. As in the case of single-phase liquid, return flows appear as a result of a pressure difference in the constituting media; however, during two-phase flow these differences are different for each of the phases. The high value of capillary pressure in slightly-permeable inclusions creates in them the zone of reduced pressure in aqueous phase. water is the simultaneously and displacing and more wetting phase. On the contrary, in the displaced phase pressure in the inclusions it is

higher than in the high-permeability medium. Because of this appears the return flow of water in the switchings on, but the displaced phases - in the opposite direction.

Page 206.

If the relation of permeability $k_1/k_2$ is very great, capillary pressure in slightly-permeable inclusions considerably more than in the high-permeability medium. then it is possible to consider the exchange of the liquid between the media because of capillary forces as result of the capillary impregnation of slightly-permeable switchings on. Therefore the intensity of overcurrents q can be determined on the basis of the investigation of countercurrent capillary impregnation.

To investigate the course of impregnation, by taking into account the true form of switchings on or assemblies, is virtually impossible; furthermore, with rare exception/eliminations the form of assemblies is unknown; therefore it is necessary to proceed from the analysis of the impregnation of linear specimen/sample.

In Chapter VI, §4 it was obtained, that when the initial saturation of $s = s_0$ was constant, then with small the saturation is the function of alternating/variable $\xi = x/a\sqrt{t}$, and the rate of filtration of each of the phases v with x = 0 in absolute value it is expressed by formula

$$v = ac(s_1, s_0)\, t^{-1/2}. \qquad \text{(VII.3.4)}$$

. In this case, the rate of the being absorbed phase is equal to +v, and that which is extracted is equal to -v.

After the approach of the front of displacement to the enclosed end/lead of the specimen/sample, the average saturation in it approaches the constant value of $s = s_1$ in accordance with approximation (VI.4.39)

$$s = s_1 (1 - A e^{-\alpha t/\tau_0}).$$

. At the same time value v decreases in the course of time exponentially

$$v = Ae^{-\alpha t/\tau_0}, \qquad \text{(VII.3.5)}$$

where A and $\alpha$ - constants, but $\tau_0 = l^2/a^2$.. For the approximate description of the course of impregnation constant A it is necessary **to select** so that rate v would be continuous at certain value of $t = t_1$, . so that **with** $t < t_1$ was valid the formula **(VII.3.4)**, and with $t > t_1$ - formula (VII.3.5).

As a whole the course of the impregnation of linear specimen/sample is illustrated by the given above in Fig. VI.22

curve/graphs of the dependence of the average saturation $\bar{s}$ on time. With small t value $\bar{s}$ linearly depends on $\sqrt{t}$, a v - inversely proportional $\sqrt{t}$. With large t $\bar{s}$, it approaches $s_1$, and v decreases exponentially.

If permeability $k_1$ is sufficiently great, then from the torque/moment of the expedition of water to slightly-permeable inclusions or the assemblies in cracked-porous medium on their boundary immediately is establish/installed a maximally possible value of saturation $s = s^*$. Under such conditions, if the initial saturation of assemblies is constant, it is possible to consider that the advance of the water, which is absorbed into assembly, and therefore the intensity of return flows depend only on time the determinations of this assembly or cell/element in the irrigated zone.

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Page 207.

Let us introduce the new unknown function $t_0$ (x, y, z) - the transit time of the front of displacement in the high-permeability medium or in the cracks through the point with coordinates x, y, z. It goes without saying that this is possible only in such a case, when it is possible to isolate this front, i.e., the surface, on the one hand of which in cracks (or in the high-permeability medium) appeared the displacing liquid, and on the other hand its saturation was equal to the initial value. Under the enumerated conditions the

intensity of return flows it is the function of time of the determination of assembly in the zone, included by the displaced liquid (irrigated). This time is equal $t-t_0$ (x, y, z) = $\tau$.

The form of the function q ($\tau$) can be selected, on the strength of expressions for the velocity of the impregnation of one cell/element (VII.3.4) and (VII.3.5).

In order to pass from the velocity of the impregnation of one cell/element to the intensity of return flows per unit of volume of the medium with dual porosity, it is necessary the velocity of impregnation v to multiply by the specific surface area of slightly-permeable assemblies and by certain coefficient, depending on the form of these assemblies. In this case, one should consider that the impregnation at the particular point of layer begins only after the approach to it of the front of displacement. specific surface area from the considerations of dimensicnality can be expressed in the form of the $\beta l^{-1}$, where $\beta$ is is constant, $l$ - the significant dimension of assembly. Then from equality (VII.3.4) we obtain that at the low values $\tau$q ($\tau$) it is possible to accept in the form:

$$q = N_1 \frac{a}{l} \tau^{-1/2} = N_1 (\tau_0 \tau)^{-1/2}, \qquad \text{(VII.3.6)}$$

where the $\tau_0 = \frac{l^2}{a^2}$ are characteristic time of the impregnation of assembly; $N_1$ - dimensionless constant. For large $\tau$ of (VII.3.5) follows the formula

$$q = N_2 \frac{a^2}{l^2} \exp\left(-\frac{\alpha\tau}{\tau_0}\right). \qquad \text{(VII.3.7)}$$

• convenient approximation for q $(\tau)$ at all values $\tau$ is the function of the form:

$$q(\tau) = A \frac{e^{-b\tau}}{\sqrt{\tau}}. \qquad \text{(VII.3.8)}$$

This function with low $\tau$ coincides with expression (VII.3.6), and with large $\tau$ it decreases in such a way that the complete soaking itself volume per unit of volume of the medium, equal $\int_0^\infty q(\tau)\,d\tau$, is final. Constant A and b in formula (VII.3.8) it is not difficult to fit so that with low $\tau$ equation (VII.3.8) would coincide with (VII.3.6), and with large $\tau$ $s^{(2)}$ it would approach the maximum saturation of assembly after impregnation $s_1$. Then

$$A=N_1\frac{a}{l};\quad b=\frac{\pi N_1^2 a^2}{l^2 m^2 s_1^2}. \qquad \text{(VII.3.9)}$$

The expression of form (VII.3.8) was suggested E. by V. Skvortsov [104].

Page 208.

The intensity of return flows it is possible to introduce into equations (VII.3.2) - (VII.3.3) and differently, after assuming that it depends on the saturation of each of the medium. In this case, it is possible to consider that q is proportional with ($s_1$, $s_2$) [formula

(VII.3.4) ]. Dependence with ($s_1$, $s_2$), as show the calculations of self-similar solutions, approximately is represented in the form:

$$c(s_1, s_2) = K(s_1)(s_1 - s_2). \qquad \text{(VII.3.10)}$$

Instead of $s_1$ and $s_2$ into expression (VII.3.10) one should supply $s^{(1)}$ and certain function of $s^{(2)}$, selected taking into account the fact that the impregnation occurs before the equalization of capillary pressures in both media. This method of the introduction of the intensity of return flows was used in V. M. Ryzhik's work [98].

For the description of displacement in the cracked medium in equations (VII.3.2) and (VII.3.3) one should place n = 1, (1 - n) $\vec{U}_i^{(1)} = \vec{w}_i$ on the basis of the fact that the volume of cracks is small in comparison with the volume of the pores of $\vec{w}_i$ they are average by entire volume of the rate of filtration of phases. After designating, further, $s^{(2)} = s$ and $m_2$ = m, we will obtain

$$\operatorname{div}\vec{w}_1 + q = 0;\quad \operatorname{div}(\vec{w}_1 + \vec{w}_2) = 0;\quad m\frac{\partial s}{\partial t} - q = 0. \quad \text{(VII.3.11)}$$

Let us examine the one-dimensional tasks of displacement in cracked-porous layer. The system (VII.3.11) will be led to the form:

$$\frac{\partial w_1}{\partial x} + q = 0;\quad m\frac{\partial s}{\partial t} - q = 0;\quad w_1 + w_2 = w(t). \quad \text{(VII.3.12)}$$

Let the initial saturation of assemblies be constant and the intensity of return flows can be expressed by the formula of form $q = q(\tau)$.

Let us integrate the first of the equations (VII.3.12) from entrance ($x = 0$) to the "front" of the invading water. taking into account that with $x = 0$ moves the only displacing phase and $w_1 = w(t)$, we will obtain

$$w(t)=\int_0^{x_0(t)} q\left(t-T(x)\right)dx. \qquad \text{(VII.3.13)}$$

From (VII.3.13) it is possible to obtain integral equation for the displacement of the front of the displacing liquid in cracks $x_0(t) = f(t)$. Let us introduce in equation (VII.3.13) the new variable of integration T, by set/assuming $x = f(T)$. We will obtain

$$w(t)=\int_0^t q(t-T)f'(T)\,dT. \qquad \text{(VII.3.14)}$$

Page 209.

After determining from integral equation (VII.3.14) function $f(t)$ or inverse function $T(x) = t_0(x)$, from the second equation of system (VII.3.12) we can find the distribution of the saturation of assemblies s in any point in time:

$$s - s_0 = \frac{1}{m} \int\limits_{T(x)}^{t} q\left(\tau - T(x)\right) d\tau = \frac{1}{m} \int\limits_{0}^{t-T(x)} q(\tau)\, d\tau. \quad \text{(VII.3.15)}$$

The right side of the equation (VII.3.14) takes the form of fold and it can be solved by the Laplace transform method. Let

$$\int\limits_0^\infty w(t)\, e^{-\sigma t}\, dt = W(\sigma); \quad \int\limits_0^\infty q(t)\, e^{-\sigma t}\, dt = Q(\sigma); \quad \text{(VII.3.16)}$$

$$\int\limits_0^\infty f(t)\, e^{-\sigma t}\, dt = \Phi(\sigma).$$

By using the theorem about fold and the initial condition f (0) = 0, we will obtain from (VII.3.14)

$$W(\sigma) = \sigma \Phi(\sigma)\, Q(\sigma). \quad \text{(VII.3.17)}$$

whence

$$\Phi(\sigma) = \frac{W(\sigma)}{\sigma Q(\sigma)}. \qquad \text{(VII.3.18)}$$

Let us examine some special cases. For zero times when q $(\tau)$ is expressed by formula (VII.3.6), we have

$$Q(\sigma) = N_1 \frac{a}{l} \sqrt{\frac{\pi}{\sigma}},$$

whence

$$\Phi(\sigma) = \frac{W(\sigma)}{N_1 \frac{a}{l} \sqrt{\pi\sigma}}. \qquad \text{(VII.3.19)}$$

Specifically, if the rate of displacement w (t) is changed according to the power law of $w = w_0 t^{\beta}$, that

$$W(\sigma) = \frac{w_0 \Gamma(\beta+1)}{\sigma^{\beta+1}}$$

and

$$\Phi(\sigma) = \frac{w_0 l \Gamma(\beta+1)}{N_1 a \sqrt{\pi}\, \sigma^{\beta+\frac{3}{2}}}; \quad f(t) = \frac{w_0 l \Gamma(\beta+1)}{N_1 a \sqrt{\pi}\, \Gamma\left(\beta+\frac{3}{2}\right)} t^{\beta+\frac{1}{2}}. \quad \text{(VII.3.20)}$$

For the case of constant velocity $\beta = 0$ and

$$f(t) = \frac{2w_0 l}{N_1 \pi a} \sqrt{t}. \qquad \text{(VII.3.21)}$$

. In $\beta < -1/2$ solution (VII.3.20) does not have the physical sense, since it does not satisfy condition f (0) = 0.

Page 210.

The complete description of displacement from the cracked-porous medium is conveniently conducted with the use of dependence q ($\tau$) in the form (VII.3.8). The function of form (VII.3.8) approximates well the course of impregnation in the linear case, and its conversion according to Laplace takes the comparatively simple form, which makes it possible to solve equation (VII.3.14) in the final form for a series of the important cases of dependence w (t).

With q (t), by the being expressed formula (VII.3.8),

$$Q(\sigma)=\frac{A\sqrt{\pi}}{\sqrt{\sigma+b}}, \qquad \text{(VII.3.22)}$$

whence

$$\Phi(\sigma)=\frac{W(\sigma)\sqrt{\sigma+b}}{A\sqrt{\pi}\,\sigma}. \qquad \text{(VII.3.23)}$$

Let us examine again case of $w(t) = w_0$ = const. Then

$$\Phi(\sigma)=\frac{w_0}{A\sqrt{\pi}}\frac{\sqrt{\sigma+b}}{\sigma^2}. \qquad \text{(VII.3.24)}$$

From the tables of the Laplace transform it is possible to find $f(t)$ in the form:

$$f(t)=\frac{w_0}{A\sqrt{\pi b}}(1+2bt)\operatorname{erf}\left(\sqrt{bt}\right)+\frac{2w_0}{A\sqrt{\pi b}}(1-e^{-bt}); \quad \text{(VII.3.25)}$$

$$f'(t)=\frac{2w_0\sqrt{b}}{A\sqrt{\pi}}\operatorname{erf}\left(\sqrt{bt}\right). \quad \text{(VII.3.26)}$$

From formula (vii.3.26) it is evident that with $t \to \infty$ the speed of the displacement of front becomes constant and equal to

$$V=\frac{2w_0\sqrt{b}}{A\sqrt{\pi}}.$$

In accordance with formula (VII.3.15) the saturation of assemblies s, if f (t) = Vt, is expressed in the form:

$$s=s_0+\frac{1}{m}\int_0^{t-\frac{x}{V}} q(\tau)\,d\tau=s_0+(s_1-s_0)\operatorname{erf}\left(\sqrt{b\left(t-\frac{x}{V}\right)}\right), \quad \text{(VII.3.27)}$$

i.e. s is function x - Vt. Thus, in the limit we obtain solution of the type, described in §3 chapter VI, i.e., solution of the type of the traveling wave. At great significance of t - x/V the saturation of assemblies approaches the constant extreme value of $s = s_1$. All the change in the saturation from $s_0$ to $s_1$ occurs in the zone whose extent is of the order of $\frac{w_0 l^2}{N_1 a^2} \cdot (2 \div 3)$. This zone by analogy with the case of displacement in the homogeneous medium (see Chapter VI, §3) was called name stabilized. Within the stabilized zone actually is realized entire process of the impregnation of assemblies.

Page 211.

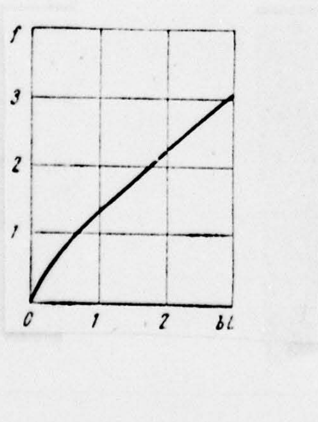



Fig. VII.6.

Dependence f (t), that corresponds to equation (VII.3.25), is shown in Fig. VII.6 [104]. It is evident that the stabilized zone is formed for time of order (1/b)•(2-3), i.e., approximately 2-3 times is more time of the impregnation of one assembly. with small t function f (t) is of the order of $\sqrt{t}$, i.e. the course of displacement is such as in the case of $q = c/\sqrt{t}$.

Exactly as the examined case of linear displacement, it is possible to investigate radial tasks. During the radial flow when the displacing liquid is forced through the hole whose center is accepted as the origin of coordinates, the first of the equations (VII.3.11) will be written in the form:

$$\frac{1}{r}\frac{\partial}{\partial r}(rw_1) + q = 0. \qquad \text{(VII.3.28)}$$

By integrating by r from r = $\rho$ (duct of the hole) to the position of front in cracks r = R, we will obtain

$$G(t) = 2\pi \int_{\rho}^{R(t)} q\left(t - T(r)\right) r\,dr, \quad \text{(VII.3.29)}$$

where $G(t) = 2\pi\rho\ w_0(t)$; expenditure/consumption of liquid through the hole per the unit of power of the layer: $t(r) = t_0(r)$ is time of the appearance of a front of liquid on circle with radius of r. let by function, reverse/inverse T (r), will be R (T). Let us accept as the unkrown function $\phi(t) = \pi R^2(t)$ - the area, included by the moving front. then from (VII.3.29) is obtained the following integral equation for $\phi(t)$:

$$G(t) = \int_0^t q(t-T)\varphi'(T)\,dT. \qquad \text{(VII.3.30)}$$

. This is accurate the same equation as (VII.3.14), only instead of the rate of filtration w (t) in it enters expenditure/consumption G (t). All the given solutions to equation (VII.3.14) can be transferred for radial flow. for determining saturation in assemblies

from (VII.3.12) we will obtain analogously (VII.3.5)

$$s - s_0 = \int_0^{t-T(r)} q(\tau)\, d\tau. \qquad \text{(VII.3.31)}$$

Let us note that in the case of G (t) = const, if q is determined from formula (VII.3.8), then from the formulas, analogous (VII.3.6) or (VII.3.18), we will obtain with large t

$$R(t) \approx C\sqrt{t}. \qquad \text{(VII.3.32)}$$

. Page 212.

The obtained solution besides the cracked-porous medium can be used also for the description of displacement in the two-layered layer when the power/thickness slightly-permeable layer is very great and $k_2/k_1$ very little, and the high-permeability layer can be considered as slot (it is analogous with the case, described in §2).

Under this condition, if we disregard impregnation in longitudinal direction, each cell/element slightly-permeable layer (cut out the perpendicular to X-axis) is impregnated according to self-similar law, i.e., $q = \frac{C}{\sqrt{\tau}}$. Solution for the displacement of front over slot retains form (VII.3.20), the only constant $N_1 \frac{a}{l}$ is replaced by C = ac [see (VII.3.4)]. If the power/thickness of slightly-permeable layer is final, then one should utilize the same expressions q ($\tau$) that and for the cracked-porous medium.

The approach outlined above to the tasks of the displacement of the nonmiscible liquids in the cracked-porous medium was developed in works by V. M. Ryzhik [98, 3], A. A. Bokserman, S. P. Zheltov, A. A. Kocheshkov, V. L. Danilov [30, 31].

Until now, we examined the tasks of the displacements in which the exchange of the liquid between cracks and assemblies (or between the sections of different permeability) it was produced by the action of capillary forces. however, under the specific conditions of the media with dual porosity, the course of displacement can substantially change also because of the unsteady processes of exchange, caused by the elastic redistribution of pressure between assemblies and cracks. This factor is utilized in the process of cyclic water-flood which is applied under conditions of sharply

heterogeneous and cracked collector/receptacles. With cyclic water-flood the consumption of the inject/begun rocking into layer water (or another liquid) periodically changes. This change causes the periodic exchange of the liquid between cracks and assemblies because of the elastic redistribution of pressure. In the course of exchange, occurs the gradual enrichment of assemblies by the displacing liquid which, obviously, faster is moved on cracks. The emerging of the assemblies liquid always therefore has less saturation as the displacing phase, than the liquid, which is located in cracks and which enters the assemblies. Capillary forces intensify this process, since the more wetting displacing liquid (water) is held in assemblies because of "end effects". But also in the case when liquids - the completely being mixed and capillary effects are absent, cyclic mode/conditions leads to the exchange of the liquids between assemblies and cracks and to the gradual extraction of the displaced liquid. the simplified diagram of the cyclic process of displacement stated below in layers with dual porosity was proposed by A. A. Bokserman and B. V. Shalimov [32].

In the simplest statement for the analysis of principle properties the cyclic process of displacement can be examined using an example of displacement from the cracked-porous layer of liquids, equal densities, viscosity and compressibility (polychromatic

liquids).

Page 213.

In these conditions the pressure field in assemblies and cracks, is described by system of equations (VII.1.11) and does not depend on the distribution of saturation.

For the system of dynamically identical liquids in question relative permeability as for flow in cracks, sc also in assemblies are equal to the appropriate saturations. The equations of the continuity of the j phase in cracks and assemblies respectively take the form:

$$\frac{\partial}{\partial t}(m_1 \rho s_j^{(1)}) + \operatorname{div}(\rho U_j) = -\rho q_j; \qquad \text{(VII.3.33)}$$
$$\frac{\partial}{\partial t}(m_2 \rho s_j^{(2)}) = \rho q_j,$$

where the $s_j^{(1)}$ - saturation in cracks; $s_j^{(2)}$ - saturation in assemblies. Are here made the same assumptions that and when deriving the equations (VII.1.11). In accordance with what has been said, it is higher than the $\vec{U}_j = s_j^{(1)}\vec{U}$, where the $\vec{U} = \vec{U}_1 + \vec{U}_2$ are the total rate of filtration in cracks.

We have:

$$q_j = \begin{cases} s^{(1)}q & q > 0; \\ s^{(2)}q & q < 0, \end{cases} \qquad \text{(VII.3.34)}$$

i.e. if return flow it goes from cracks into assemblies, then the overflowing liquid has the same composition, as liquid in cracks, and vice versa. If we accept function q according to formula (VII.1.5), then of the system (VII.3.33) can be obtained system (VII.1.11) and, furthermore, the following equations for the saturation:

$$\left.\begin{aligned} m_1 \frac{\partial s^{(1)}}{\partial t} + U \operatorname{grad} s^{(1)} &= 0 \\ m_2 \frac{\partial s^{(2)}}{\partial t} &= (s^{(1)} - s^{(2)})\, q \end{aligned}\right\} q > 0; \qquad \text{(VII.3.35)}$$

$$\left.\begin{aligned} m_1 \frac{\partial s^{(1)}}{\partial t} + U \operatorname{grad} s^{(1)} &= (s^{(1)} - s^{(2)})\, q \\ m_2 \frac{\partial s^{(2)}}{\partial t} &= 0 \end{aligned}\right\} q < 0. \qquad \text{(VII.3.36)}$$

Equations (VII.3.35) and (VII.3.36) show that in that of the media, from which occurs the return flow, saturation is not changed in accordance with the assumption that the composition of the overflowing liquid the same as and in that medium, whence occurs return flow (actually under the effect of capillary forces in the emerging from assemblies liquid will predominate the displaced phase, that accelerate the process of exchange).

If we examine the displacement of the nonmiscible liquids taking into account the true form of relative permeability, then saturation will be changed with any sign q.

In the formulation of the problem in question the pressure fields and saturation can be determined in turn means to the solution of equations (VII.1.11) and (VII.3.35) - (VII.3.36).

Page 214.

For further simplification in the task, one must take into account that the return flow of liquid from crack into assembly and back for time of one cycle is small in comparison with the total volume of liquid in assembly, since this return flow occurs only because of compressibility. Therefore a change in the saturation occurs considerably slower than pressure change. This makes it possible to average all parameters in equations (VII.3.35) and (VII.3.36) for time interval, equal to one period, i.e., to introduce the averaged variables of the form:

$$\langle s^{(i)} \rangle = \frac{1}{T} \int_0^T s^{(i)}\, dt. \qquad \text{(VII.3.37)}$$

By considering deviations from the average values small, we will obtain for the average values of saturation system of equations

$$m_1 \frac{\partial \langle s^{(1)} \rangle}{\partial t} + \langle U \rangle \operatorname{grad} \langle s^{(1)} \rangle = 0;$$
$$m_2 \frac{\partial \langle s^{(2)} \rangle}{\partial t} = (\langle s^{(1)} \rangle - \langle s^{(2)} \rangle)\, q_0, \qquad \text{(VII.3.38)}$$

where

$$q_0 = \frac{1}{2T} \int_0^T |q|\, dt.$$

In order to determine the form of the function $q_0$ (x), one should find the appropriate periodic solutions of equation (VII.1.11) and average them for time. For the case of the one-dimensional flow between galleries we will search for the periodic solutions of equation (VII.1.11) with period of $T = 2\pi/\omega$:

$$\frac{\partial p_1}{\partial t} - \eta \frac{\partial^3 p_1}{\partial t\, \partial x^2} = \varkappa \frac{\partial^2 p_1}{\partial x^2}. \qquad \text{(VII.3.39)}$$

Solution we search for in the form:

$$p_1 = p_0(x) + P_1^0 e^{\alpha x} e^{i\omega t} \qquad \text{(VII.3.40)}$$

so that it would satisfy an equation (VII.3.39) and periodic boundary conditions

$$\frac{k_1 h}{\mu_1}\frac{\partial p_1}{\partial x}\Big|_{x=0} = Q_0 + Q_1\cos(\omega t+\delta_1);$$
$$\frac{k_1 h}{\mu_1}\frac{\partial p_1}{\partial x}\Big|_{x=l} = Q_0 + Q_2\cos(\omega t+\delta_2). \qquad \text{(VII.3.41)}$$

From equation (VII.3.39) it follows that $p_0''(x) = 0$ and $p_0(x) = A + Bx$. Value A is unessential constant, and

$$B = \frac{\mu_1 Q_0}{k_1 h}.$$

Page 215.

Substituting expression (VII.3.40) in equation (VII.3.39), we obtain

$$\alpha^2 = \frac{\omega^2\eta + i\omega\varkappa}{\varkappa_1^2 + i\omega^2\eta^2}, \qquad \text{(VII.3.42)}$$

whence are determined two root: $\alpha_1$ and $\alpha_2$.

Set/assuming $\alpha_1 = \gamma_1 + i\gamma_2,\ \alpha_2 = \gamma_1 - i\gamma_2,$ we have

$$p_1(x,\ t) = p_0(x) + [C_1 e^{i\gamma_2 x} + C_2 e^{-i\gamma_2 x}]\, e^{\gamma_1 x + i\omega t} = p_0(x) + P_0(x)\, e^{i\omega t}. \qquad \text{(VII.3.43)}$$

By substituting expression (VII.3.43) in condition (VII.3.41) and by isolating real part, it is possible to find the values of the constants $C_1$ and $C_2$. After are found function $p_1$ (x, t), the form of function of return flow it can be determined from the second equation (VII.1.11), i.e.,

$$p_1 - p_2 = \varkappa \frac{\partial^2 p_1}{\partial x^2} = \frac{\varkappa}{A} \frac{d^2 P_0}{dx^2} e^{i\omega t}. \qquad \text{(VII.3.44)}$$

The determination of constants in the given solution is sufficiently cumbersome and we will not bring final formulas.

As showed the numerous calculations, carried out by A. A. Bokserman and B. V. Shalimov [32], at the usual values of periods -

the order of several hours - pressure amplitude in practice it does not depend on coordinate x. Consequently, and the amplitude of the intensity of return flows $q_0$ can be accepted as independent of x. For the constant $q_0$ the solution of system (VII.3.38) in the one-dimensional case can be obtained in the locked form under the initial condition of $\langle s^{(2)} \rangle = 0$. In the one-dimensional case of equation (VII.3.38) it is possible to write, by omitting the signs of averaging:

$$m_1 \frac{\partial s^{(1)}}{\partial t} + U \frac{\partial s^{(1)}}{\partial x} = -(s^{(1)} - s^{(2)})\, q_0;$$
$$m_2 \frac{\partial s^{(2)}}{\partial t} = (s^{(1)} - s^{(2)})\, q_0. \qquad \text{(VII.3.45)}$$

Let us introduce new independent variables

$$\xi = x; \quad \tau = t - \frac{x m_1}{U} = t - \frac{x}{V}. \qquad \text{(VII.3.46)}$$

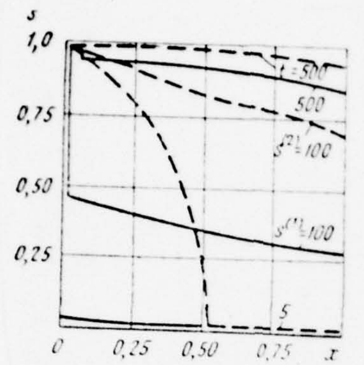
Then instead of (VII.3.45) we obtain system

$$\frac{\partial s^{(1)}}{\partial \xi} + (s^{(1)} - s^{(2)}) \frac{q_0}{V} = 0;$$
$$\frac{\partial s^{(2)}}{\partial \tau} - (s^{(1)} - s^{(2)}) \frac{q_0}{m_2} = 0. \qquad \text{(VII.3.47)}$$

System (VII.3.47) must be solved under boundary conditions

$$s^{(1)} = 1 \quad \text{при } \xi = 0 \text{ и } s^{(2)} = 0 \quad \text{при } \tau = 0. \qquad \text{(VII.3.48)}$$

Fig. VII.7.



By applying the Laplace transform for $\tau$ and by taking into account the second condition (VII.3.48), we will obtain instead of (VII.3.47) the equation

$$\frac{dS^{(1)}}{d\xi}+(S^{(1)}-S^{(2)})\,\alpha=0;$$
$$\sigma S^{(2)}-(S^{(1)}-S^{(2)})\,\beta=0, \qquad \text{(VII.3.49)}$$

where the $S^{(1)}$ and the $S^{(2)}$ - the Laplace transform from $s^{(1)}$ and the $s^{(2)}$; $\sigma$ is the parameter of the Laplace transform; $\alpha = q_0/V$; $\beta = q_0/m_2$.

from boundary conditions it follows that with $\xi=0\ S^{(1)}=\frac{1}{\sigma}$.

The solution to equations (VII.3.4) takes the form:

$$S^{(1)}=\frac{\exp(-\alpha\xi)}{\sigma}\exp\left(\frac{\alpha\beta\xi}{\alpha+\beta}\right);$$
$$\text{(VII.3.50)}$$
$$S^{(2)}=\frac{\beta\exp(-\alpha\xi)}{\sigma}\exp\left(\frac{\alpha\beta\xi}{\sigma+\beta}\right).$$

We utilize a formula

$$\int_0^\infty I_0(\sqrt{2at})\,e^{-st}\,dt = e^{\frac{a}{2s}} \qquad \text{(VII.3.51)}$$

(see [43], formula (9.3.42)]. By using formula (VII.3.51) and product rule for the Laplace transform, we will obtain the following expressions for an $s^{(1)}$ and the $s^{(2)}$:

$$s^{(1)} = \exp(-\alpha\xi - \beta\tau) I_0 (2\sqrt{\alpha\beta\xi\tau}) + \\ + \beta \exp(-\alpha\xi) \int_0^\tau \exp(-\beta\lambda) I_0 (2\sqrt{\alpha\beta\xi\lambda})\, d\lambda;$$

$$s^{(2)} = \beta \exp(-\alpha\xi) \int_0^\tau \exp(-\beta\lambda) I_0 (2\sqrt{\alpha\beta\xi\lambda})\, d\lambda. \qquad \text{(VII.3.52)}$$

From formulas (VII.3.52) it follows that at the front of the advance of water in cracks (with $\tau = 0$) $s^{(1)}_{[\tau]}$ has a jump whose

intensity is equal $\exp(-\alpha\xi)$.

The curve/graphs of the dependence of $s^{(1)}$ and $s^{(2)}$ on x with different values of t are given in Fig. VII.7.

Pages 217-254.

# Chapter VIII.

# NONLINEAR UNSTEADY FILTRATION.

## §1. Deviations from the law of Darcy. Nonlinear filtration.

In all examined, until now, tasks we assumed to be carried out the law of darcys. Is explained this by the fact that the law of darcys sufficiently accurately describes the fundamental circle of filtration motions. at the same time in a number of cases, the nonlinearity of the law of filtration becomes essential, and sometimes also determining.

As the basis of the conclusion/derivation of the law of darcys in chapter I, §2, were placed two basic assumptions: 1) the motion is inertia-free (" creeping"); 2) liquid viscous Newtonian, not interacting with the solid skeleton of the porous medium [1].

FOOTNOTE [1]. I.e. the effect of skeleton prorounces only in the fact that on its surface is satisfied the common for a viscous fluid condition of adhesion. At the same time it was assumed that the skeleton does not create the acting on liquid force field, does not adsorb any noticeable part of the liquid, it does not form with it colloid etc. ENDFOOTNOTE.

The subsequent refinements are connected with failure of these assumptions.

1. As can be seen from the very conclusion/derivation of the law of darcys (see Chapter I, §2), it must be disrupted in range sufficient high rates at which already one must not fail to consider the inertia component of the resistance to motion of liquid.

By adding to the number of determining parameters (I.2.2) density with dimensionality $ML^{-3}$, we will obtain already six values from which it is possible to form three dimensionless combinations. Repeating reasonings chapter I, §2, we obtain

$$\operatorname{grad} p = -\frac{\mu}{d^2}\vec{u} f\left(\frac{u\rho d}{\mu}\right)$$

(appearing here combination $ud\rho/\mu$ plays the role of Reynolds number of filtration micromovement).

Page 218.

Allow/assuming the possibility of the expansion of function f in Taylor series and being limited to the first two members of expansion, we obtain

$$\operatorname{grad} p = -\frac{\mu}{k}\vec{u} - \beta\frac{\rho u}{\sqrt{k}}\vec{u} \qquad \text{(VIII.1.1)}$$

(is here taken into account, that $d^2 \approx k$, see chapter I, §2).

Expression (VIII.1.1) is called of the bincmial law of filtration. For the first time binomial law was proposed by Forkheimer [117].

As show experiments, this simple expression it describes well observational data. Along with the frequently cited data of Fencher, Lewis and Burns [115], let us note another the experiments of Lindquist, reproduced in work [149]. These experiments show that

relationship/ratio (VIII.1.1) it is something larger, rather than simple empirical formula, since it is made well even for very great significance of rate of filtration. The physical sense of this entails the fact that at high rates the rapidly changing motion in pores is conjugate/combined with the advent of considerable inertia components of hydraulic resistance.

To the explanation of the physical sense of relationship/ratio (VIII.1.1) is dedicated a series of works from which it is necessary to note Ye. M. Minskiy's works [82-84].

The appearance of a quadratic term in the equation of the law of filtration sometimes is connected with the agitation of flow. However, already the exponent of Reynolds (1-10), calculated from diameter is coveted or the pores of the porous medium with which pronounce the deviations from linearity, it indicates the inaccuracy of this affirmation [149, 126]. Recently the absence of turbulence (i.e. the fluctuations of rate in time) is proved also by direct/straight experiments [157].

In the tasks of the theory of the filtration (in difference, for

example, from the tasks of chemical technology) of the application/appendix of binomial law, filtration is limited mainly by the motion of gas near high-yield gas wells or by motion near holes in the cracked media. In the latter case the special importance has that fact that true airspeed of liquid in cracks is considerably more rate of filtration.

2. The binomial law of filtration (VIII.1.1) considers deviations from the law of darcys at high rates. Another character bears the refinement, examine/considered below. We will examine the only inertia-free motions. Let us assume that besides the forces of liquid resistance there are also resisting forces whose value does not depend on rate of filtration (although it depends on its direction; resisting force are always directed against the speed of relative displacement).

Page 219.

The simplest case of system with such properties is the non-Newtonian ductile plastic liquid for which shearing stresses are connected with the gradient of speed du/dn by Bingham's relationship/ratio [85-86]:

$$\tau = \tau_0 + \mu \frac{du}{dn} \qquad \left(\frac{du}{dn} > 0\right);$$
$$\tau \leqslant \tau_0 \qquad \left(\frac{du}{dn} = 0\right). \qquad \text{(VIII.1.2)}$$

. In this relationship/ratio, besides viscosity $\mu$, enters also constant $\tau_0$, called the initial shear stress.

We allow now, that occurs the filtration of the liquid, characterized by two constants: by viscosity $\mu$ and by characteristic stress $\tau_0$. From dimensional considerations, just as in chapter I, §2, we obtain

$$\mathbf{grad}\, p = -\frac{\mu \vec{u}}{d^2} f\left(\frac{\tau_0 d}{\mu u}\right). \qquad \text{(VIII.1.3)}$$

Let us assume that the driving ir the porous medium liquid possesses that property, that with an increase in the deformation rate of deformation the value of viscous deformation becomes predominating. Then at an increase in the rate of filtration the effect of the parameter $\tau_0$ must asymptotically decrease. This means that function f must have the final limit with the tendency of argument tc zero:

$$f(0)=d^2/k, \qquad \text{(VIII.1.4)}$$

Constant k is the permeability of the medium in usual sense. Let now the filtering liquid possess that property (inherent, for example, in ductile plastic liquids), that at the low speeds of deformation the stresses do not depend cn velocity and they do not vanish with a decrease in the velocity of shift/shear to zero. It is obvious that at small rates of filtraticn in expression (VII.1.3) the rate must disappear. This means that

$$f \approx b\,\frac{\tau_0 d}{\mu u} \qquad (u \to 0). \qquad \text{(VIII.1.5)}$$

. As a result during deceleration cf filtration to zero, the pressure gradient approaches the final (ncnzerc) limit:

$$(\mathrm{grad}\ p)_0 = -\gamma = -\frac{b\tau_0}{d}. \qquad \text{(VIII.1.6)}$$

. This extreme value $\gamma$ determines that value of pressure gradient, upon achieving which begins the moticn of liquid; at the smaller values of gradient, the motion is absent. Value $\gamma$ is called limiting (initial) gradient; if for the case in question this extreme value exists, then they speak about filtration to maximum (initial) gradient. The simplest form of the law cf filtration with maximum gradient is obtained on the assumption that function f it is represented by binomial expression

$$f=\frac{d^2}{k}+b\frac{\tau_0 d}{\mu u}, \qquad (VIII.1.7)$$

to the satisfying relationship/ratios (VIII.1.4) and (VIII.1.6).

Page 220.

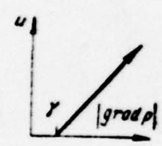



Fig. VIII.1.

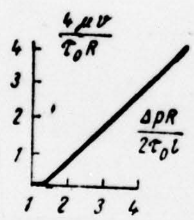



Fig. VIII.2.

In this case, equation (VIII.1.3) gives (Fig. VIII.1)

$$\begin{aligned} \operatorname{grad} p &= -\frac{\mu}{k}\vec{u} - \gamma\frac{\vec{u}}{u} \qquad (u > 0); \\ |\operatorname{grad} p| &\leqslant \gamma \qquad (u = 0). \end{aligned} \qquad \text{(VIII.1.8)}$$

The law of filtration with maximum (initial) gradient was utilized in hydraulics [96, 116] and in oil-field mechanics, first of all in A. Kh. Mirzadzhanzade's works with colleagues [85, 86, 112].

It is must, however, to note that the relationship/ratio (VIII.1.8) not necessarily is fulfilled accurately even for the filtration of ductile plastic liquid. This is easily explicable. The total resisting force is composed of the resisting forces, which act in separate pores. In this case, in each fluid element, the relationship/ratio between "the viscous" ($\mu$ du/dn) and "the plastic" ($\tau_0$) by stress components depends not only on the value of the average speed (rate of filtration), but also from the redistribution of the rates between separate pore channels.

A similar phenomenon occurs during the motion of ductile plastic liquid in one capillary (here redistribution occurs between the separate layers of liquid). As a result communication/connection between the jump/drop in the pressure and the average speed for a capillary with a radius of R takes the form (Fig. VIII.2):

$$v=\frac{\Delta p}{l}\frac{R^2}{8\mu}\left[1-\frac{4}{3}\left(\frac{2\tau_0}{R}\frac{l}{\Delta p}\right)+\frac{1}{3}\left(\frac{2\tau_0 l}{\Delta pR}\right)^4\right]. \quad \text{(VIII.1.9)}$$

Motion in capillary ceases with the pressure differential

$$\Delta p=\frac{2\tau_0 l}{R}. \quad \text{(VIII.1.10)}$$

Motion in capillary ceases with the pressure differential

Asymptote to the linear section of curve $\Delta p/l - v$ intersects with axis $\Delta p/l$ when $\Delta p = {}^8/_3 \tau_0 l/R$.

let now we we have the porous which consists of many microcapillaries of different radii. During a reduction in the pressure differential, begins the gradual "sealing" of capillaries. In accordance with formula (VIII.1.10) at first the motion ceases in the smallest capillaries, and with a decompression, occurs sealing ever larger to large capillaries.

Page 221.

It is clear that the more powerful the spread of the size/dimensions of pores, that more is expanded transition to the complete cessation

of motion and the greatly differs the true form of the law of filtration from the idealized relationship/ratio (VIII.1.8).

However, this expression can make also asymptotic sense, describing motion at relatively high rates of filtration ($u \gg \tau_0 d/\mu$). With this understanding the law of filtration with maximum gradient (VIII.1.8) describes the broad class of nonlinear filtration motions. In this case, it is logical to distinguish the true maximum gradient $\gamma_0$, which corresponds the complete cessation of motion, and the maximum gradient $\gamma$, which corresponds the asymptotic section of the law of filtration. We have by order of value

$$\gamma_0 \approx \frac{\tau_0}{d_{\text{макс}}}; \quad \gamma \approx \frac{\tau_0}{d}, \qquad \text{(VIII.1.11)}$$

where $d_{\text{макс}}$ and d - respectively maximum and medium size of pore channels. For the media with strongly heterogeneous structure, these values can be distinguished many times.

As already it was established/installed from the dimensional

considerations, $d \approx c\sqrt{k}$. Therefore for the media of the uniform structure of $\gamma \approx \tau_0/\sqrt{k}$. This relationship/ratio established/installed and experimentally was proven by B. I. Slutanov [112] (in his experiments of value $\gamma_0$ and $\gamma$ they were not distinguished).

3. Let us illustrate the aforesaid by some experimental data. Figure VIII.3a-c gives data on the filtration: a are water in clay [132]; b - water in argillized sandstone [162]; c are oil in sand [1]. As can be seen from curve/graphs, expression (VIII.1.8) describes sufficiently well motion in all these cases in the area of comparatively high velocities; at small velocities different systems behave differently.

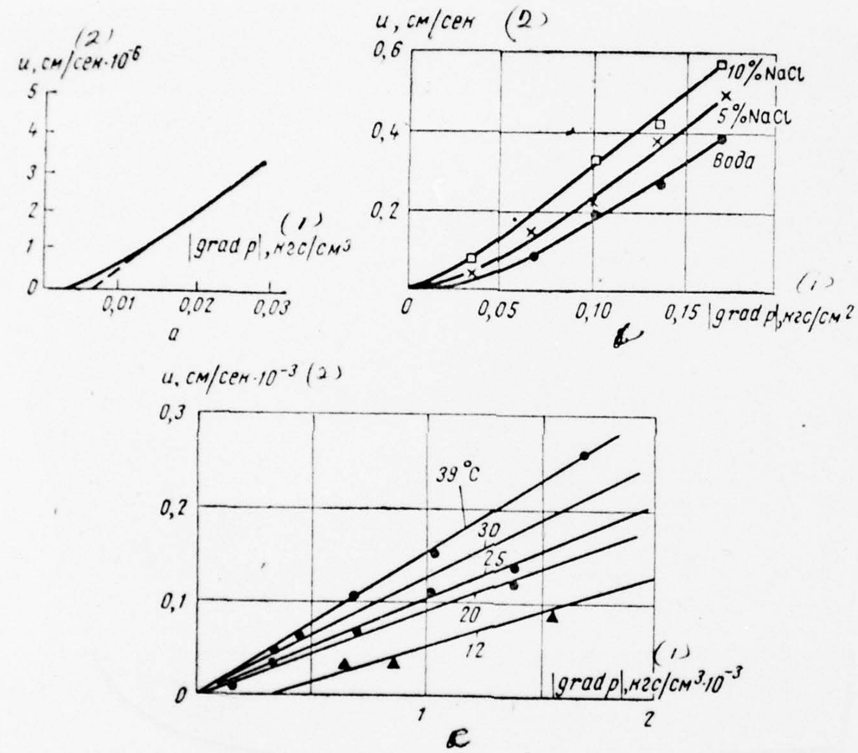
On the basis of the nonlinear behavior of systems at these cases, lie/rest different physical mechanisms. It is important, however that these effects are exhibited at the low speeds of filtration and in the media with a small size/dimension of pores (grains), i.e., with small permeability. The relative role of nonlinear effects is determined by the parameter

$$S^{\cdot}=\frac{\tau d}{\mu u} \text{ или } S^{*}=\frac{\tau \sqrt{k}}{\mu u}. \qquad \text{(VIII.1.12)}$$

This fact determines the special feature/peculiarities of nonlinear filtration in heterogeneous layers. The areas of small permeability prove to be the areas of the maximum manifestation of nonlinear effects, which contributes to the supplementary difficulty of motion in these areas. Let us examine this in an example of fine/thin laminar layer. This in practice this case will lead us also to some new formulations of the problems.

Fig. VIII.3.

Key: (1). kgf/cm³. (2). cm/s.



4. Fine/thin laminar layer by the power of $h_i$ is complex N by seams by the permeability of $k_i$. Let us assume that for each seam is valid the law of filtration with maximum gradient

$$\vec{u} = -\frac{k_i}{\mu}\left(\operatorname{grad} p - \gamma_i \frac{\operatorname{grad} p}{|\operatorname{grad} p|}\right);$$
$$(|\operatorname{grad} p| > \gamma_i);$$
$$u = 0 \quad (|\operatorname{grad} p| < \gamma_i). \qquad \text{(VIII.1.13)}$$

Let us assume that the seams are numbered in the ascending order in the permeability. In accordance with what has been said above this it means also that the values of $\gamma_i$ decrease with an increase in number i.

Let us refine now, which is understood by fine/thin layer. We will consider layer fine/thin so, that it is possible to disregard pressure change in thickness (thereby it is assumed that the

appearing between separate layers pressure differences rapidly are equalized because of the exchange of the liquid between seams).

Page 223.

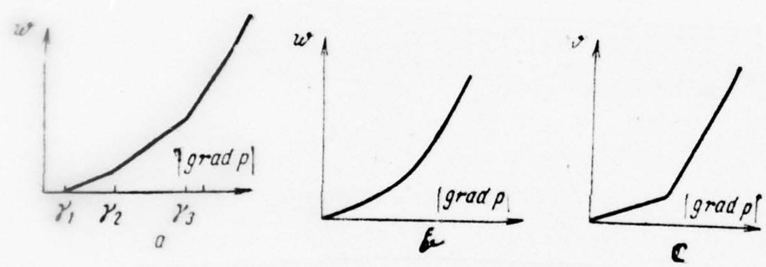



Fig. VIII.4.

Let us determine under this hypothesis the vector of $\vec{w}$ - the average rate of filtration of the liquid through the area/site whose height/altitude is equal to thickness of layer, and width is equal to unity. We have a

$$\vec{w}=\frac{1}{H}\int_0^H \vec{u}\,dz=-\frac{1}{H}\left[\sum_{i=1}^{j}\frac{k_i h_i}{\mu}\,\mathrm{grad}\,p-\sum_{i=1}^{j}\gamma_i\frac{\mathrm{grad}\,p}{|\mathrm{grad}\,p|}\right]. \quad \text{(VIII.1.14)}$$

Here the number of the $0\leqslant j\leqslant N$, to which is conducted the addition, is determined by obvious condition

$$\gamma_j<\mathrm{grad}\,p\leqslant\gamma_{j+1}. \quad \text{(VIII.1.15)}$$

With $|\mathrm{grad}\,p|<\gamma_1$ we have w = 0. Not difficult to see that the motion of homogeneous liquid in laminar layer it is possible to examine just as motion in uniform layer at the rate of filtration of $\vec{w}$ and in accordance with the law of filtration (VIII.1.14) (Fig. VIII.4a). Thus, along with the law of filtration the maximum gradient (VIII.1.8) it makes sense to examine the piecewise linear laws of

filtration, described by convex downward broken line (Fig. VIII.4a). It is hence not difficult to pass to being continuously changed according to the thickness of layer of permeability. In this case, is obtained the law of filtration, described by arbitrary convex downward curve (Fig. VIII.4b). Finally, examining the simplest case of the two-layered layer in which one of the layers possesses negligible maximum gradient, we obtain the simple piecewise-linear law of filtration (Fig. VIII.4c), in which it is possible to trace a series of the special feature/peculiarities of the movement of liquid in laminar layers. In all enumerated cases the law of filtration in velocity high area it has rectilinear asymptotic section.

In oil-field literature usually divide the cases of nonlinear filtration and disconnection of the separate layers of layer with a change in the pressure gradient. This separation, apparently, is irrational: a change in the effective power of layer not only it is one of the manifestations of the nonlinearity of the law of filtration, but also it can be described by that mathematical apparatus.

Page 224.

Subsequently we will not this specify especially, but one should remember that everything said below about nonlinear filtration allow/assumes direct interpretation in connection with motion in laminar fine/thin layers.

5. By supplementing the equation of the law of filtration by the equation of continuity and by the equation of state of liquid, we will obtain the nonlinear theory of elastic mode/conditions, or the theory of the nonlinear filtration of gas, in the same way as this it was made in chapter II.

Along with the examined piecewise linear approximation of the nonlinear law of resistance, frequently is encountered also approximation by the exponential expression of the form:

$$\vec{u} = -f(|\operatorname{grad} p|)\operatorname{grad} p, \qquad \text{(VIII.1.16)}$$

where f is exponential function:

$$f(|\operatorname{grad} p|) = C|\operatorname{grad} p|^{\alpha}. \qquad \text{(VIII.1.17)}$$

At first this approximation was utilized for the description of the law of filtration resistance in the transition region between the linear and quadratic law of resistance; in this case $\alpha < 0$. Subsequently, however, this approximation almost everywhere yielded the place of the binomial approximation, examined above. In the last/latter time the power law of filtration again acquires independent value, since it describes well the motion of a series of non-Newtonian liquids, including of solutions and polymer melts, in the porous medium. For such liquids the characteristically "pseudo-plastic" behavior when the effective viscosity of liquid falls in proportion to the increase in the deformation rate of deformation and index $\alpha$ is positive.

§2. Self-similar solutions of the problem of the filtration of liquid in the nonlinear law of resistance. Filtration with maximum gradient.

The self-similar solutions of the problems of filtration with the nonlinear law of resistance, if it is not exponential, exist only during the narrow special selection of the initial and boundary conditions. However, self-similar solutions are important in that relation, that they make it possible to explain the special feature/peculiarities of the appearing nonlinear tasks.

1. the rectilinear-parallel motion of elastic liquid. Piecewise linear law of filtration. Let us examine filtrational motion under the nonlinear law of resistance in the conditions of elastic mode/conditions. Is expressed velocity from the equation of the law of filtration through the pressure gradient in the form:

$$\vec{u} = -\frac{k\gamma}{\mu}\Psi\left(\frac{|\operatorname{grad} p|}{\gamma}\right)\frac{\operatorname{grad} p}{|\operatorname{grad} p|}. \qquad \text{(VIII.2.1)}$$

Page 225.

Here k is permeability of the medium; $\mu$ - the viscosity of liquid; $\gamma$ - the characteristic value of pressure gradient; $\Psi$ are the dimensionless function, which describes the law of filtration.

During this notation it is assumed that at the law of filtration has the linear section for which it is possible to determine ratio $k/\mu$ (it is obvious, this requirement is carried out for all examined in §1 laws of filtration, except exponential). By substituting expression (VIII.2.1) in the equation of continuity (II.1.3) and by counting liquid and layer elastic-deformed, let us arrive at the equation of elastic mode/conditions under the law of filtration (VIII.2.1):

$$\frac{\partial p}{\partial t} = \varkappa\gamma \operatorname{div}\left[\Psi\left(\frac{|\operatorname{grad} p|}{\gamma}\right)\frac{\operatorname{grad} p}{|\operatorname{grad} p|}\right]. \qquad \text{(VIII.2.2)}$$

. Here $\varkappa$ - the piezoconductivity, calculated by usual method from the above-determined values k and $\mu$:

$$\varkappa=\frac{kK}{m\mu}, \qquad \text{(VIII.2.3)}$$

a value K and m they make usual sense.

Specifically, for one-dimensional [redacted] motion in direction of x-axis we have

$$\frac{\partial p}{\partial t}=\varkappa\gamma\frac{\partial}{\partial x}\Psi\left(\frac{1}{\gamma}\frac{\partial p}{\partial x}\right). \qquad \text{(VIII.2.4)}$$

Let be examined the semi-infinite layer and the initial distribution of pressure in it linearly, but not to the boundary of layer are supported constant selection or the pumping of liquid, so that

$$\left.\frac{\partial p}{\partial x}\right|_{x=0}=B;\quad p(0,\ x)=Ax. \qquad \text{(VIII.2.5)}$$

It is easy to ascertain that the task with such conditions is self-similar and has solution of the form:

$$p=\gamma x f(\xi);\quad \xi={}^{1}/_{2}x/\sqrt{\varkappa t}. \qquad \text{(VIII.2.6)}$$

For function f, is obtained the equation

$$-2\xi^{2}\frac{df}{d\xi}=\frac{d}{d\xi}\,\Psi\left(f+\xi\frac{df}{d\xi}\right) \qquad \text{(VIII.2.7)}$$

under the conditions

$$f(\infty)=\frac{A}{\gamma}=\alpha;\quad \lim_{\xi\to 0}\left(f+\xi\frac{df}{d\xi}\right)=\frac{B}{\gamma}=\beta. \qquad \text{(VIII.2.8)}$$

We will examine the piecewise linear law of filtration (see Fig. VIII.4c):

$$\begin{aligned}&\Psi(y)=\varepsilon y \qquad (|y|<1;\quad \varepsilon<1);\\&\Psi(y)=\varepsilon\,\mathrm{sgn}\,y+y-\mathrm{sgn}\,y \qquad (|y|>1).\end{aligned} \qquad \text{(VIII.2.9)}$$

Page 226.

There is special interest in the limiting case of $\varepsilon\to 0,$ when is obtained the law of filtration with the initial gradient, examined above.

Under the law of the filtration of form (VIII.2.9) the presented task can be solved in an explicit form. Equation (VIII.2.7) falls into two linear equations:

$$\begin{aligned} &\xi f'' + 2(1+\xi^2) f' = 0 \qquad (|f+\xi f'| > 1); \\ &\varepsilon\xi f'' + 2(\varepsilon+\xi^2) f' = 0 \qquad (|f+\xi f'| < 1). \end{aligned} \qquad \text{(VIII.2.10)}$$

The range of values of the argument of $0<\xi<\infty$ is divide/marked off on several sections of $(l_i, l_{i+1})$ in such a way that for each of them is made one of the equations (VIII.2.10), whereupon on adjacent sections solution satisfies different equations. The number and the character of the location of sections is easy to establish/install from the considerations of continuity, if one considers that the solutions to equations (VIII.2.10) and their derivatives are monotonic.

1. If gradients A and B (we will call them initial and final respectively) - one sign and in absolute value more the critical

gradient of $\gamma\,(|\alpha|>1, |\beta|>1,\ \alpha\beta>0)$, that in all layer gradient exceeds critical in absolute value. On entire direct/straight $0<\xi<\infty$ is made the first equation (VIII.2.11), and task is reduced to known linear task.

2. If the initial and final gradients on model is less than critical, then with entire straight line is made the second equation (VIII.2.10), and task again is reduced to linear.

3. If $|\alpha|>1, |\beta|<1,$ then on the adjoining the boundary of layer section of the $[0, l]$, where $l$ - the unknown boundary, to be determined, pressure gradient less than critical, and is made the second equation; in interval $(l, \infty)$ - the first.

4. If $|\alpha|<1, |\beta|>1,$ then with the section $[0, l]$ near the boundary of layer is made the first equation (VIII.2.10), and with remaining half-line - the second.

5. Finally, if the initial and final gradients, exceeding on module/modulus critical, they have the different signs of

$(|\alpha|>1, |\beta|>1, \alpha\beta<0)$, then the range of motion is divide/marked off into three sections: on $(0, l_1)$ and $(l_2, \infty)$ satisfies the first equation (VIII.2.10), but on $(l_1, l_2)$ - the second.

For each section solution it can be extracted in an explicit form:

$$f(\xi)=C_1+D_1\left[\sqrt{\pi}\operatorname{erf}\xi+\xi^{-1}\exp(-\xi^2)\right];$$
$$f(\xi)=C_2+D_2\left[\sqrt{\pi}\operatorname{erf}(\xi/\sqrt{\varepsilon})+\xi^{-1}\sqrt{\varepsilon}\exp(-\xi^2/\varepsilon)\right] \quad \text{(VIII.2.11)}$$

- respectively for the first and second equations (VIII.2.10). Therefore in order to achieve the purpose, sufficiently to find constant C and D the boundary of $l_i$ for all sections. Equations for constants are obtained from supplementary conditions (VIII.2.8) and the conditions of coupling on boundaries. These conditions consist in the fact that the pressure and consumption are continuous on the boundaries of section, whereupon the position of these boundaries is determined from the supplementary requirement for the equality of gradient critical.

Page 227.

Thus, we have:

$$f(l+0)=f(l-0); \quad (f+\xi f')_{l+0}=(f+\xi f')_{l-0}=\pm 1. \quad \text{(VIII.2.12)}$$

. The sign under the last/latter condition (VIII.2.12) is determined from the considerations of continuity. It is not difficult to check that the conditions (VIII.2.12) together with boundary conditions give as much equations, as unknowns has in task.

Let us examine now consecutively those three of that which were enumerated above five possibilities with which the task does not transfer/convert to linear. For case 3 the solution, which satisfies boundary conditions (VIII.2.5), takes the form:

$$f(\xi)=\beta-D_2\left[\sqrt{\pi}\,\mathrm{erf}\,(\xi/\sqrt{\varepsilon})+\xi^{-1}\sqrt{\varepsilon}\exp(-\xi^2/\varepsilon)\right] \qquad (0\leqslant\xi\leqslant l);$$
$$f(\xi)=\alpha+D_1\sqrt{\pi}-D_1\left[\sqrt{\pi}\,\mathrm{erf}\,\xi+\xi^{-1}\exp(-\xi^2)\right] \qquad (l<\xi<\infty) \qquad \text{(VIII.2.13)}$$

a from conditions (VIII.2.12) it follows the system of algebraic equations for $D_1$, $D_2$ and $l$:

$$\alpha+D_1\sqrt{\pi}\,\mathrm{erfc}\,l=\mathrm{sgn}\,\alpha;$$
$$\beta-D_2\sqrt{\pi}\,\mathrm{erf}\left(\frac{l}{\sqrt{\varepsilon}}\right)=\mathrm{sgn}\,\alpha;$$
$$D_1 e^{-l^2}=D_2\sqrt{\varepsilon}\,e^{-\frac{l^2}{\varepsilon}}. \qquad \text{(VIII.2.14)}$$

System (VIII.2.14) as is easy to verify that is unambiguously solved. Analogously, in the case of 4 the solution is determined by expressions

$$f(\xi) = \beta - D_1\left[\sqrt{\pi}\,\mathrm{erf}\,\xi + \xi^{-1}e^{-\xi^2}\right] \quad (0 \leqslant \xi \leqslant l);$$
$$f(\xi) = \alpha + D_2\sqrt{\pi} - D_2\left[\sqrt{\pi}\,\mathrm{erf}\left(\frac{\xi}{\sqrt{\varepsilon}}\right) + \xi^{-1}\sqrt{\varepsilon}\,e^{-\frac{\xi^2}{\varepsilon}}\right] \quad (l < \xi < \infty), \tag{VIII.2.15}$$

in which $D_1$, $D_2$ and $l$ are determined from system

$$\begin{gathered}\alpha + D_2\sqrt{\pi}\,\mathrm{erfc}\left(\frac{l}{\sqrt{\varepsilon}}\right) = \mathrm{sgn}\,\beta;\\ \beta - D_1\sqrt{\pi}\,\mathrm{erf}\,l = \mathrm{sgn}\,\beta;\\ D_2\sqrt{\varepsilon}\,e^{-\frac{l^2}{\varepsilon}} = D_1 e^{-l^2}.\end{gathered} \tag{VIII.2.16}$$

Let us now move on to the limit with of $\varepsilon \to 0$. In this case, in accordance with expression (VIII.2.9) the law of filtration

accepts in the limit the form:

$$\Psi(y)=0 \qquad (|y|\leqslant 1);$$
$$\Psi(y)=y-\operatorname{sgn} y \qquad (|y|\geqslant 1); \qquad \text{(VIII.2.17)}$$

corresponding to filtration with maximum gradient, when the motion of liquid begins only on the achievements of the critical pressure gradient.

Page 228.

This law of filtration is degenerated and for it the very formulation of the problem with the assignment of the subcritical value of pressure gradient is doubtful. As we shall see further on, the correct formulation of the task of filtration with the initial gradient requires in order that on boundary of the region would be assigned not pressure gradient, but rate of filtration.

Let us examine at first case of 3. From system (VIII.2.14) it

follows that the $l \to 0$ with of $\varepsilon \to 0$. Actually, let $l$ it remains more $\delta > 0$. Then $D_2$ remains that which was limited, also, from the last/latter equation of $D_1 \to 0$, which contradicts the first equation (VIII.2.14).

In order to understand the sense of the obtained result, let us examine a following special case of the dismantled task. Let the initially stationary filtration flow, pressure gradient in which exceed critical, instantly it is stopped as a result of the fact that the pressure gradient on boundary becomes less than critical. It is possible to be to expect that near boundary appears the stagnation zone in which the motions nc, and then, with removal/distance from it, the rate of motion it will gradually increase. Solution shows that this not thus. There is no stagnation zone, and motion occurs in all layer, whereupon the rate of motion monotonically decreases with the approach to the boundary of layer and becomes zero on boundary itself, so that pressure gradient is here equal critical.

The pressure distribution in the range of motion is defined by the second equation (VIII.2.13), in which necessary to rely

$$D_1 = (\operatorname{sgn} \alpha - \alpha)/\sqrt{\pi}), \qquad \text{(VIII.2.18)}$$

as this is obtained from the formula (VIII.2.14) with $l = 0$. It is not difficult to see that this result does not depend on which was given the value of pressure gradient on boundary ($\beta$). As an example Fig. VIII.5 gives the results of solutions for $\beta$ = 0, $\alpha$ = 2 and three values of $\varepsilon$ $[p^\circ = \dot{\xi} f(\xi)]$.

In the case of 4, transition to the limit with of $\varepsilon \to 0$, obviously, gives the limited field of motion, which adjoins the boundary of layer, whereupon the remaining part of the layer is occupied with stagnation zone. This character of solution is perceived also directly from expressions (VIII.2.15) and the system (VIII.2.16), in which it is possible to pass to the limit with of $\varepsilon \to 0$, by considering $l$ final. Solutions for $\beta$ = 2, $\alpha$ = 0 and three values of $\varepsilon$ are given in Fig. VIII.6.

In the case of 5, range of motion is divide/marked off into three zones, and for determining unknown boundaries and coefficients of equations it is necessary to examine the system six equations, analogous in its structure for systems (VIII.2.14) and (VIII.2.16). By transfer/converting in this system to the limit with of $\varepsilon \to 0$, it is possible to ascertain that the range, in which the pressure gradient less than critical, decreases to zero, so that in the limit layer is divide/marked off into two parts in which the motion occurs towards each other. Pressure distribution for the case $\alpha = 2$, $\beta = -2$ it is shown in Fig. VIII.7.

Page 229.

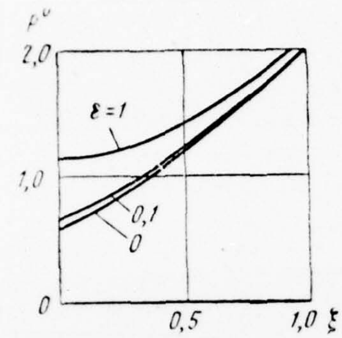



Fig. VIII. 5.

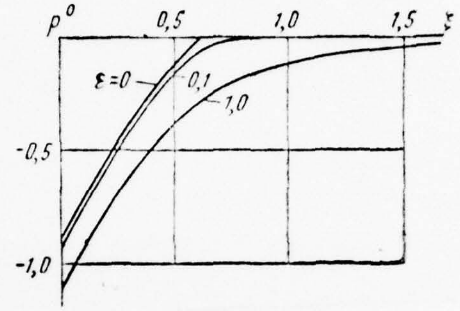



Fig. VIII. 6.

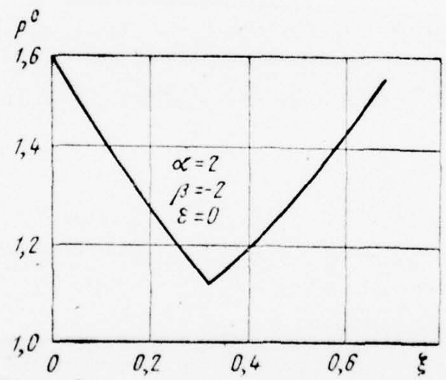



Fig. VIII. 7.

Everything obtained in the limit with of the $\varepsilon \to 0$ of distribution have point of inflection in the place of the seam of two solutions. If motions initially was not (case 4), then the disturbance/perturbation, introduced by boundary condition, covers only the final, being enlarged in the course of time region of layer. therefore the solution, obtained for an infinite layer, is applicable without any reservations and to the layer of the finite dimensions for the limited time intervals. But if motion in layer existed from the very beginning (cases 3 and 5), then disturbance/perturbation instantly covers entire layer as in the appropriate tasks of elastic mode/conditions.

2. Axisymmetrical one-dimensional motion under conditions of elastic mode/conditions. Power law of filtration. In the axisymmetrical case the motion is self-simulating under the arbitrary law of filtration, if is examined the task of the launching/starting of the bore hole of a negligible radius with the debit, which is changed proportional to square root from the time, calculated off the torque/moment of the launching/starting of bore hole

$$Q(t)=A\sqrt{t}. \qquad (VIII.2.19)$$

Page 230.

At this law of a change in the debit of bore hole it is possible to find the solution, in particular, in the law of filtration with maximum gradient [49]. However, the artificiality of the formulation of the problem lowers its practical interest, and in this case it is necessary to be limited to approximate solutions. At the same time under the power law of filtration (VIII.1.17) axisymmetrical motion is self-simulating during a change in the debit of bore hole after launching/starting in the arbitrary power law:

$$Q(t) = Qt^{\beta}. \quad \text{(VIII.2.20)}$$

In most common form these types of motion are examined in work [5]; below we will limit ourselves to the basic for application/appendices case $\beta = 0$, which answers the instantaneous launching/starting of bore hole with the constant debit Q. We have for distribution pressures p (r, t) taking into account (VIII.1.17) equation

$$\frac{\partial p}{\partial t} = a^2 \frac{1}{r} \frac{\partial}{\partial r}\left[r\left(\frac{\partial p}{\partial r}\right)^{\alpha+1}\right]; \quad a^2 = \frac{KC}{m}, \quad \text{(VIII.2.21)}$$

which must be solved under the conditions

$$p(r, 0)=0; \quad -\left[r\left(\frac{\partial p}{\partial r}\right)^{\alpha+1}\right]_{r=0}=\frac{Q}{C}. \qquad \text{(VIII.2.22)}$$

It is not difficult to see that the solution of problem self-simulating and it it is possible tc search for in the form:

$$p=At^{\frac{\alpha}{3\alpha+2}}f(\xi); \quad \xi=r\left(\frac{C}{Q}\right)^{\frac{\alpha}{3\alpha+2}}(a^2t)^{-\frac{\alpha+1}{3\alpha+2}};$$
$$A=\frac{1}{(3\alpha+2)^{1/\alpha}}\left(\frac{Q}{C}\right)^{\frac{\alpha+2}{3\alpha+2}}a^{\frac{2\alpha}{3\alpha+2}}. \qquad \text{(VIII.2.23)}$$

Substituting (VIII.2.23) in (VIII.2.21), we obtain equation

$$\frac{d}{d\xi}\{\xi[f'(\xi)]^{\alpha+1}\}+\xi[(\alpha+1)\xi f'(\xi)-\alpha f(\xi)]=0. \qquad \text{(VIII.2.24)}$$

Equation (VIII.2.24) must be solved under the conditions

$$\lim_{\xi\to 0}\xi[f'(\xi)]^{\alpha+1}=\mathrm{const}=(3\alpha+2)^{\frac{\alpha+1}{\alpha}}. \qquad \text{(VIII.2.25)}$$

Further, it is obvious, that with $\xi\to\infty$ $(r\to\infty)$ function f ($\xi$) must approach 0 itself together with its derivatives, whereupon in such a way that integral

$$M=\int_0^\infty \xi f(\xi)\,d\xi \qquad \text{(VIII.2.26)}$$

would remain finite.

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THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS (TEORIY--ETC(U)
JAN 77 G I BARENBLATT, V M YENTOV, V M RYZHIK

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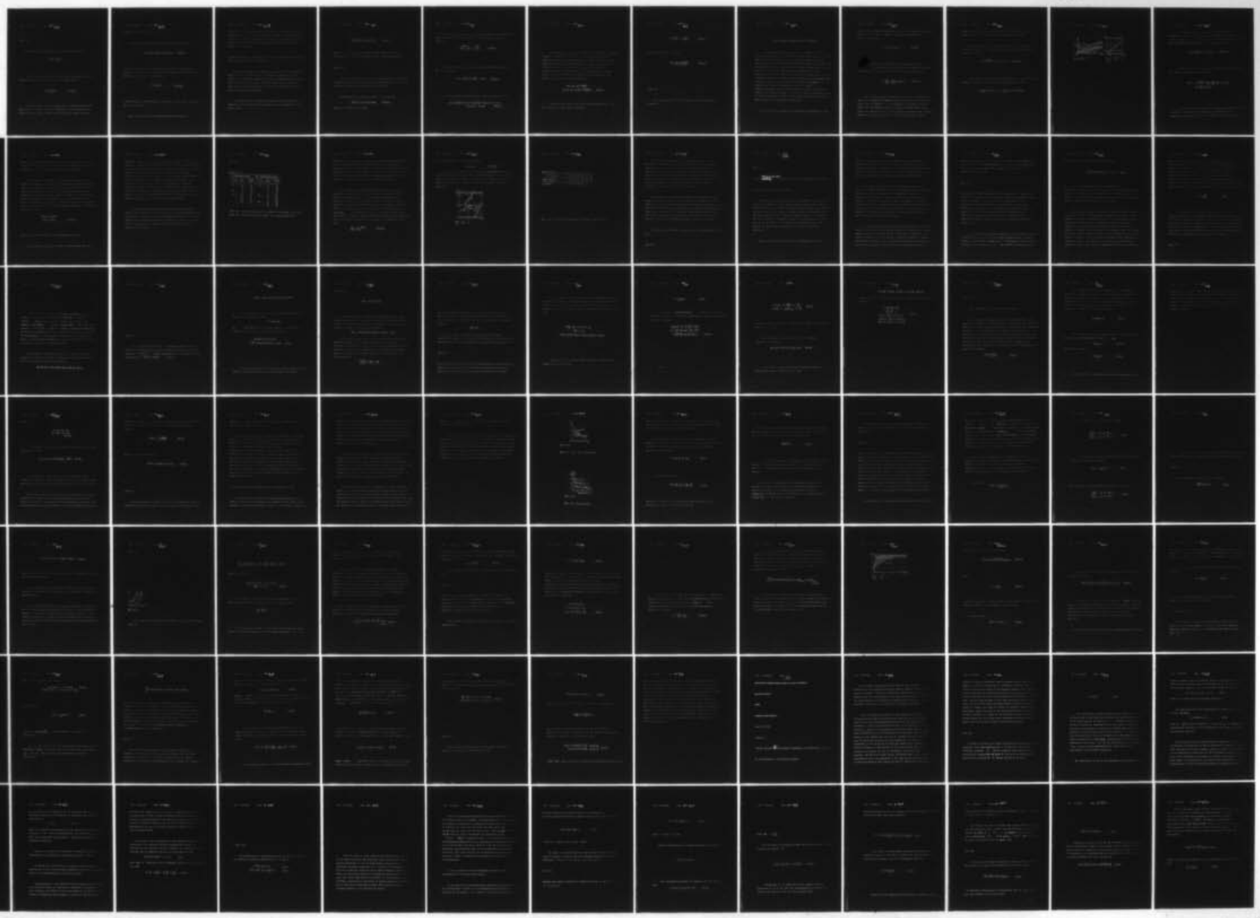

Page 231.

From condition (VIII.2.25) it follows that with $\xi \to 0$

$$f'(\xi) \approx -B\xi^{-1/(\alpha+1)}.$$

With $\alpha > 1$ (case which us now interests) derivative f' ($\xi$) is integrated near $\xi = 0$, so that there is a final limit

$$f(0) = \lim_{\xi \to 0} f(\xi). \qquad \text{(VIII.2.27)}$$

This means that under the power law of filtration (VIII.1.17) with index $\alpha > 0$ makes sense the formulation of the problem of the selection of the liquid through the bore hole of a zero radius at

final pressure in bore hole.

Further from (VIII.2.27) and (VIII.2.23) it follows that

$$p_c = p(0,\ t) = Af(0)\,t^{\beta};\quad \beta = \alpha/(3\alpha + 2), \qquad \text{(VIII.2.28)}$$

i.e. the pressure in bore hole changes according to power law, whereupon the exponent of this law $\beta$ is unambiguously connected with the degree of the law of filtration $\alpha$:

$$\beta = \alpha/(3\alpha + 2), \qquad \text{(VIII.2.29)}$$

a coefficient A is proportional to the debit of bore hole Q to degree $(\alpha + 2) / (3\alpha + 2)$.

Thus, the obtained self-simulating solution offers the

possibility of the experimental determination both index $\alpha$ and the coefficient C of the law of filtration. In more detail about this it is said in §3, where the law of filtration is represented in the form of the dependence of pressure gradient on rate of filtration and index $n = 1/(\alpha + 1)$.

§3. The approximate solution of the tasks of nonstationary filtration in the nonlinear law of resistance.

Above have already been examined the simplest (self-simulating) motions with the nonlinear law of filtration. However, the most important for application/appendices task of axisymmetrical inflow to the bore hole, released with a constant debit, under conditions of the nonlinear law of filtration is non-self-simulating; below this task is examined approximately.

1. Assuming that the filtering liquid and layer elastic are compressed, let us write down the basic system of equations of task in the form:

$$\frac{\partial p}{\partial r} = \gamma\Phi\left(\frac{q}{rU}\right); \quad \frac{\partial p}{\partial t} = \frac{K}{m}\,\frac{1}{r}\,\frac{\partial q}{\partial r}. \qquad \text{(VIII.3.1)}$$

Here $q = ru$; $U = \gamma k/\mu$ is a characteristic value of rate of filtration; $\gamma$ - the characteristic value of pressure gradient.

Page 232.

Equations (VIII.3.1) they must be solved under the additional conditions, which correspond the launching/starting of bore hole in the layer in which initially the motions was not.

Assume/taking the initial pressure as zero, we have

$$p(r,\,0) = 0; \quad q(a,\,t) = Q = \text{const.} \qquad \text{(VIII.3.2)}$$

Here a, a radius of bore hole,.

Let us accept at first for the law of filtration the expression, which corresponds filtration with the initial gradient (see Fig. VIII. 1):

$$|\Phi(j)| \leqslant 1 \qquad (j=0);$$
$$\Phi(j) = j + \operatorname{sgn} j \qquad (j \neq 0). \qquad \text{(VIII.3.3)}$$

Sufficient accuracy is reached, if we draw nearer function q (r, t) by parabola

$$q(r,\ t) = Q\left(1 + \frac{br}{l} + \frac{cr^2}{l^2}\right) \qquad (r \leqslant l). \qquad \text{(VIII.3.4)}$$

the corresponding expression for a pressure takes the form:

$$p(r,\ t) = \frac{\gamma Q}{U}\left[\ln\frac{r}{l} + b\left(\frac{r}{l} - 1\right) + \frac{1}{2}c\left(\frac{r^2}{l^2} - 1\right)\right] + \gamma(r - l), \quad (r \leqslant l);$$
$$p\ (r, t) = 0, \qquad (r \geqslant l). \qquad \text{(VIII.3.5)}$$

From condition q (l, t) it follows to c =-1-b. For determining remaining unknown b (t) and l (t) we use two first integral correlations, that are obtained from the second equation of system (VIII.3.1) after its multiplication by r and r² respectively and integration from 0 to . In this case, in all intermediate calculations, we disregard ratio a/ in comparison with unit/one (those same we are limited to examination only sufficient long times). We have after simple computations

$$Ql^2(3+b)+4Ul^2=\frac{24KUQt}{\gamma m};$$
$$\frac{d}{dt}\left[l^3(8+3b)+15\frac{Ul^4}{Q}\right]=\frac{30KUl\,(4+b)}{\gamma m}. \qquad \text{(VIII.3.6)}$$

Excluding from system (VIII.3.6) alternating/variable b (t) and introducing dimensionless variables

$$\lambda = \frac{1}{12}\frac{U^2 l^2}{Q^2}; \quad \tau = \frac{KU^3}{\gamma m Q^2}\, t, \qquad \text{(VIII.3.7)}$$

we obtain first order equation

$$\frac{d\lambda}{d\tau} = \frac{10\tau - 7\lambda + 40\sqrt{3}\,\lambda^{3/2}}{6\tau - 3\lambda + 24\sqrt{3}\,\lambda^{3/2}}. \qquad \text{(VIII.3.8)}$$

.

Page 233.

The unknown solution to this equation must satisfy obvious conditions

$$\lambda(0)=0;\quad \lambda(\infty)=\infty;\quad \lambda'(\tau)\geqslant 0 \quad \text{при} \quad \tau>0. \quad \text{(VIII.3.9)}$$

Condition (VIII.3.9) they separate the unique solution of equation (VIII.3.8). Let us note first of all, that point (0.0), past which must pass solution, he is for equation (VIII.3.8) person. Investigating it on first approximation (for example, see [87]), easy to ascertain that this saddle, and of its two separatrices, a and b, which pass in the first quadrant, have tangents with angular coefficients of 1 and 10/3. The second of these separatrices as it is possible to verify, it leaves the first quadrant in the second, and separatrix with the initial angular coefficient of $(d\lambda/d\tau)_{\tau=0}=1$ satisfies all conditions (VIII.3.9). This is easy to establish/install, using a phase diagram of equation (VIII.3.8), shown in Fig. VIII.8. Indices 0 and ∞ here noted the isoclinal lines of zero and infinity [on which becomes zero respectively numerator and denominator of expression (VIII.3.8)].

Let us find for function λ (τ) asymptotic expression for great

significance of time. If one assumes that $\lambda(\tau)$ grow/rises faster or slower than the $\tau^{2/3}$, then of (VIII.3.8) we will obtain contradiction. Hence

$$\lambda(\tau)=c\tau^{2/3}+o(\tau^{2/3}). \qquad \text{(VIII.3.10)}$$

Substituting this expression in (VIII.3.8), we find

By continuing the process of the sequential isolation of terms, it is possible to obtain the more exact expression:

$$\lambda=\frac{\tau^{2/3}}{\sqrt[3]{48}}-\frac{\tau^{1/3}}{4\sqrt[3]{36}}+\frac{1}{192}+o(1). \qquad \text{(VIII.3.11)}$$

The initial section of dependence $\lambda(\tau)$ can be found by the numerical integration of equation (VII.3.8) taking into account the known value of $\lambda'(0)=1$. The comparison of calculated thus the values with the values, given by asymptotic formula (VIII.3.11), shows that already with $\tau$ - 1 this formula is accurate with accuracy several percentages. Dependence $\lambda(\tau)$ is shown in Fig. VIII.9

[initial section of curve is obtained by numerical integration; beginning with $\tau = 1$ - according to formula (VIII.3.11) ].

By knowing $\lambda(\tau)$, it is possible to find b and c from the initial equations. From (VIII.3.6) after passage to dimensionless variables we have

$$b=\frac{2\tau-8\sqrt{3}\lambda^{3/2}}{\lambda}-3;\quad c=-1-b. \qquad \text{(VIII.3.12)}$$

With small $\tau$ from $\lambda(\tau)\approx\tau$ it follows to $b\approx-1$, $c\approx 0$. With large $\tau$, using an asymptotic formula (VIII.3.10), we have

$$b=\frac{3}{4\sqrt[3]{6\tau}}+o(\tau^{-1/3});\quad c=-1-\frac{3}{4\sqrt[3]{6\tau}}+o(\tau^{-1/3}). \qquad \text{(VIII.3.13)}$$

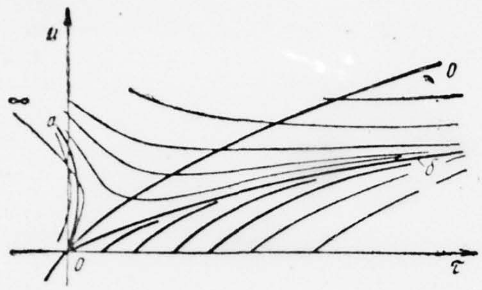


Fig. VIII. 8.

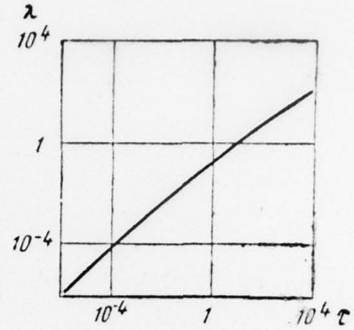


Fig. VIII. 9.

If is examined pressure change for sufficiently great significance of time in the point, close to bore hole, so that $r/l \ll 1$, $rU/Q \ll 1$, then formula for pressure distribution after the rejection of low terms can be presented in the form:

$$p(r,\ t) = \frac{\gamma Q}{U}\left(\ln\frac{r}{l} - b - \frac{c}{2}\right) - \gamma l. \qquad \text{(VIII.3.14)}$$

Substituting here the values of coefficients and returning to the initial variables, we have finally

$$p(r,\ t) = -\sqrt[3]{\frac{6\gamma^2 KQt}{m}} - \frac{\gamma Q}{3U}\ln\frac{6KQt}{\gamma m r^3} + \frac{\gamma Q}{6U} \qquad \text{(VIII.3.15)}$$
$$\left(t \gg \frac{\gamma m r^3}{KQ};\ \frac{rU}{Q} \ll 1\right).$$

2. With the aid of the method of integral correlations it is possible to find also the solution of the problem of the launching/starting of bore hole in nonlinear to the zone of the

filtration of the form:

$$\varphi(j) = (j/n)^n \quad (j \leqslant n);$$
$$\varphi(j) = j - n + \operatorname{sgn} j \quad (j \geqslant n), \qquad \text{(VIII.3.16)}$$

which in the limit with of $n \to 0$ it passes to the law of filtration with maximum gradient. In this case the pressure in bore hole p (a, t) changes in time according to the law

$$p(a,\ t) \approx -\text{const}\cdot t^{\frac{1-n}{3-n}}\cdot Q^{1-\frac{2(1-n)}{3-n}} + \ldots \qquad \text{(VIII.3.17)}$$

3. Let us use the now obtained results to an analysis of some data on a study of bore holes for nonstationary inflow.

Let us make the at first following observation. Let be examined a change of pressure p (t) in certain fixed point of the layer, the motion of liquid in which is caused by the launching/starting of hole with a constant debit (specifically, can be examined pressure in bore

hole itself).

Page 235.

Layer we will consider uniform and not limited. Let us select certain time interval $\Delta t$ and examine value

$$\Delta p(t)=p(t+\Delta t)-p(t). \qquad \text{(VIII.3.18)}$$

. Then a change in value $\Delta p$ (t) at large t depends exclusively on the form of the law of filtration in the area of low speeds (the more lesser, the the more times in question). This assertion is sufficiently obvious: in the adjacent to bore hole area the motion is stabilized and pressure distribution turns out to be the same as during steady motion (see also chapter III and IV). As a result the value of this area in the deviation of pressure in bore hole from the initial level stops to change and no longer shows up in value $\Delta p$. That area in which occurs the basic rearrangement of flow and which gives the basic contribution to value $\Delta p$ (t), turns out to be that

which was moved away from bore hole. The greater the time t, that further this area and fact less therefore amounted to here rates of filtration. By this is proven the formulated assertion.

Let us examine now inflow to bore hole under conditions of nonlinear filtration, but with that additional condition that at the rates of filtration less than certain $u < u_0$, the law of filtration can be approximately represented to straight line (as in Fig. VIII.4a, b, c). Then, as shown above, the nature of pressure change in bore hole with long times will be the same as in the case of linear filtration. Specifically, pressure change will be proportional to the debit of bore hole Q and will linearly depend on the logarithm of time, so that

$$\Delta p(t) \approx -Q \ln \frac{t+\Delta t}{t};$$
$$p(t) \approx C - Q \ln t. \qquad \text{(VIII.3.19)}$$

(in this case value itself C can nonlinear depend on Q).

Let us assume now that we analyze a series removed with the

different debits Q of the curves of pressure change in bore holes. Then with sufficiently long times all the curves in coordinates $\Delta p$ - ln t it will have straight portions, according to which in the usual way will be determined the hydroconductivity of layer $kh/\mu$, obviously, which does not depend on Q (at the same time the value of the led radius of the bore holes of aaaaaaa will prove to be, possibly, that depends on Q). On the other hand, a change in hydroconductivity $kh/\mu$ with change Q indicates the nonlinearity of the law of filtration at small rates (in this case we will call the law of filtration substantial-nonlinear or norlinearized; the case, examined above, let us call that which is linearized).

4. Let us examine from this viewpoint the data [56] (Table VIII.1), which clearly show an increase in the hydroconductivity with an increase of debit. It is especially important that this effect is distinctive as pressuring (where it it would be possible to explain by crack opening with an increase in the pressure), so also to operational bore holes.

Page 236.

Table VIII.1.

| (1) Нагнетательные скважины | | | (2) Эксплуатационные скважины | | |
|---|---|---|---|---|---|
| № скважины (3) | Дебит, $м^3$/сутки (4) | Гидропроводность, д·см (5) | № скважины (3) | Дебит, м/сутки (4) | Гидропроводность, д·см (5) |
| 514 | 793 | 838 | 1529 | 153 | 951 |
| | 757 | 839 | | 140 | 950 |
| | 540 | 760 | | 85 | 875 |
| | 360 | 693 | | 65 | 832 |
| 1246 | 745 | 575 | | 40 | 670 |
| | 511 | 485 | | 23 | 620 |
| | 323 | 314 | | 16 | 473 |
| 367 | 1364 | 1111 | 1608 | 216 | 3200 |
| | 1333 | 1007 | | 208 | 2910 |
| | 1182 | 964 | | 154 | 2350 |
| | 1106 | 984 | | 93 | 1700 |
| | 742 | 676 | | 51 | 1070 |
| | 561 | 567 | 1288 | 149 | 843 |
| | 553 | 527 | | 115 | 770 |
| | | | | 101 | 730 |
| | | | | 88 | 555 |

Key: (1). Injection wells. (2). Operaticnal bore holes. (3). bore hole. (4). Debit, twenty-four hours. (5). Hydroconductivity.

According to that which was stated above, these data indicate action on the stratified conditions nonlinearized laws of filtration. Moreover, such kind nonlinearity is manifested not directly near bore hole (where the nonlinear effects of a change in the effective power of layer are revealed with direct observation [29]), but far from bore hole, in the range of the slow movement of liquid.

Let us attempt to now quantitatively describe the observed nonlinearity on the basis of those examined in paragraphs 1 and 2 model laws of filtration. In this case, we will proceed from assumption about the fact that the data [56] correctly reflect the dependence of a pressure increment $\Delta p$ for the comparable intervals of time on the debit of bore hole $Q$ (so that the observed value of $kh/\mu \approx Q/\Delta p$; for greater detail, see [46]). At the same time, assuming that occurs the law of filtration (VIII.3.3) or (VIII.3.16), it is possible to establish that it answers the dependence of the form:

$$\frac{(kh)_1}{(kh)_2} \approx \left(\frac{Q_1}{Q_2}\right)^{\frac{2(1-n)}{3-n}}. \qquad \text{(VIII.3.20)}$$

n = 0 corresponding to the law (VIII.3.3).

$$0 \leqslant j \leqslant {}^2/_3. \qquad \text{(VIII.3.21)}$$

Let us present now data Tables VIII.1 in coordinates lg kh - lg Q (Fig. VIII.10). According to the formula (VIII.3.20) of the points, which correspond the specific bore hole, they must lie/rest on one straight line with the angular coefficient of j, which satisfies inequality

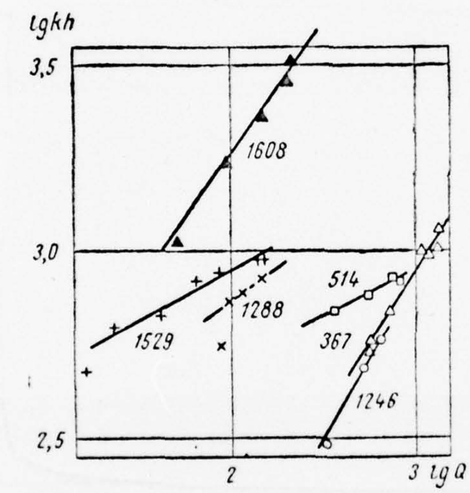


Fig. VIII.10.

Table VIII.2.

| (1) № скважины | 1608 | 1529 | 1288 | 514 | 367 | 1246 |
|---|---|---|---|---|---|---|
| Угловой коэффициент (2) . . . . . . . . | 0,67 | 0,28 | 0,36 | 0,24 | 0,74 | 0,84 |
| Показатель $n$ (3) . . . . . . . . . . . . | 0 | 0,68 | 0,56 | 0,73 | — | — |

Key: (1). bore hole. (2). Angular coefficient. (3). Index n.

The angular coefficients of straight lines, given in Fig. VIII.10, are shown in Table VIII.2 together with the corresponding them values of n. As can be seen from table, the results of calculation for bore holes 367 and 1246 indicate inexplicable rapid increase in the hydroconductivity with an increase in the debit. It is possible, this is connected with the fact that for both these bore holes (pressuring) the permeability can also increase with an increase of pressure on the face of bore hole.

For remaining bore holes are obtained the physically allowed values of index n. However, these results on no account are final. From them it is possible to make only one reliable conclusion: in the investigated speed range of filtration, the effective permeability of layer with a decrease in the velocity monotonically diminishes to zero (with the accuracy in question). It is logical, this damping filtration it can have fundamental value it needs a deep study.

The results presented were obtained in V. M. Yentova's work [45, 46].

Page 238.

Chapter IX.

## SPECIAL [redacted] PROBLEMS IN THE NONSTATIONARY FILTRATION OF HOMOGENEOUS LIQUID.

### 1. Nonisothermic filtration motion.

until now, it was assumed that the temperature of liquid or gas during their motion in porous medium remains constant. This idea is connected with the fact that temperature changes, which appear during pressure change in the course of motion, largely are compensated for by heat exchange with the skeleton of porous medium. the majority of the tasks in which is examined a change in the temperature in layer as a result of pumping in the layer of heat-transfer agent, for example hot water, can be related to the theory of stationary filtration.

However, even small a change in the temperature can be in

principle measured and used for an analysis of their causing hydrodynamic processes [121]. Therefore we will examine here the common/general/total formulation of the problem of the nonisothermic motion of compressible liquid and let us give solution for the case of the launching/starting of bore hole with a constant debit.

1. Rejecting requirement of isothermicity, we must add to a number of unknown functions even the temperature of liquid. It would be possible, using an equation of conservation of energy in connection with liquid, to compose equation for the temperature of liquid. In this case,, however, we would clash with the need for considering the heat exchange, proceeding between the liquid and its containing porous medium. The description of this heat exchange requires the study, which considers the laws governing motion and heat exchange in separate pore channels.

In order to avoid the connected with this difficulties, we will use the fact that the heat exchange between the liquid and the porous medium occurs over enormous surface (or, which is equivalent, the depth of warming up is very small), so that the existing in certain torque/moment difference in the temperatures between the liquid and the skeleton disappears very rapidly. The characteristic time of

temperature balance comprises by order of value of the $l^2/a$, where $a$ is coefficient of thermal diffusivity by least heat-conducting from the contacting media, and $l$ is a significant dimension of the structure of pore space.

Page 239.

Set/assuming $l \approx 10^{-1}$ cm., $a \approx 10^{-3}$ cm²/s, we have a $l^2/a \approx 10$ s. Thus, the recovery time of temperature differences (second and less) it is incomparable less than the times, characteristic for filtration. Therefore, without being interested in short-term effects, we will consider the temperatures of liquid and skeleton equal. As a result at each point, will be defined the united temperature, introduced and all the other characteristics of filtration flow, by means of averaging on elementary macrovolume (comp. §I.1).

2. Let us isolate now certain volume of V porous medium, limited by surface of S, and let us compose for it the equation of energy balance. By designating through the $\varepsilon$ the specific internal energy of liquid, and through the $\varepsilon_1$ - the specific internal energy of

the substance of solid skeleton, we will obtain

$$\frac{\partial}{\partial t}\int_V [em\rho+\varepsilon_1\rho_1(1-m)]\,dV-W+Q=0. \qquad \text{(IX.1.1)}$$

Here the volumetric integral determines the full/total/complete reserve of energy in the chosen volume (kinetic energy on the strength of its smallness it is disregarded); Q is a quantity of heat, bearable through surface of S; W is work, produced with the external forces above the given volume.

Heat transfer through the boundary is caused by two mechanisms. The first of them - the mechanism of thermal conductivity - is not connected with the displacement/movement of the macroscopic volumes (not separate molecules) of substance; the second mechanism is convective - it determines heat transfer with the macrovolumes of substance. However, this separation is excessively rough. In a number of cases, typical examples of which are the turbulent motion and the motion of liquid in porous medium, is possible certain intermediate situation, when in heat transfer are essential, besides thermal agitations and transfer with the average/mean motion, even transfer

with random smallscale deviations from the average/mean motion. In the examination of some scme neutralized mcticn characteristics alone, it is convenient to unite all fluctuation mechanisms under one common/general/total term - thermal conductivity. In order to emphasize difference from the usual (molecular) mechanism of transfer, are introduced the concepts of convective diffusion and thermal conductivity in porous medium. The heat flow because of this united mechanism of thermal conductivity is determined by expression

$$q_i = -\lambda_{ij}\frac{\partial T}{\partial x_j}. \qquad \text{(IX.1.2)}$$

. The presence in the equation (IX.5.2) of the tensor coefficient of thermal conductivity instead of cne scalar coefficient cf thermal conductivity is connected with the fact that the process of thermal conductivity during filtraticn possesses anisotropy, since the tensor of thermal conductivity no longer is the tensor material, the constants of liquid, but depends also on the characteristics of filtration flow (first of all from rate of filtration).

Page 240.

This completely is analogous with the fact which occurs during convective diffusion [3, 58]. However, unlike the case of diffusion, the coefficient of the thermal conductivity of the saturated porous medium comparatively barely depends on rate of filtration [47]. Therefore sufficient accuracy is reached when using the simplest correlation

$$q_i = -\lambda \frac{\partial T}{\partial x_i}. \qquad \text{(IX.1.3)}$$

Taking into account that which was stated above, we can write down

$$Q = \int_S (\vec{u}\rho e + \vec{q})\vec{n}\,dS. \qquad \text{(IX.1.4)}$$

. Here the first term expresses convective of energy transfer with

the average/mean motion.

During the determination of work of the external forces W, we will not consider gravitational force, since further will not meet the tasks in which its value is substantial, then everything it will come to the calculation of work of the forces, which act over surface of S. In this case, it occurs that one should consider only work of normal stresses (pressure), and work of shearing stresses can be disregarded. Actually, work of forces of pressure per unit time it comprises

$$W^{\bullet} = -\int_S p\vec{n}\vec{u}\,dS = -\int_V \nabla(p\vec{u})\,dV = -\int_V \vec{u}\,\nabla p\,dV - \int_V p\nabla\vec{u}\,dV.$$

During the filtration of the incompressible fluid of $\nabla\vec{u} = 0.$ In this case,, using a law of the filtration of Darcy of $\nabla p = -\frac{\mu}{k}\vec{u},$ we obtain

$$W^{\bullet} \approx \frac{\mu u^2 V}{k}.$$

At the same time for work of shearing stresses $\tau$, we have the following simple estimate/evaluation: $W'' \approx \tau u S$. But the $\tau \approx \mu u/l$, where $l$ is a significant dimension of pore space. Therefore $W'' \approx \mu u^2 S/l$, so that $W''/W' \approx kS/lV$. But $k \approx l^2$ (in actuality even $k \ll l^2$), ratio V/S - order L (significant dimension of the range in question). Thus, according to the basic assumptions of $W''/W' \approx l/L \ll 1$. For gas this calculation is not applicable, since value W' is close to zero, but also in this case it is possible to ascertain that W' is negligibly small.

Substituting the obtained expressions in equation (IX.1.1), passing from surface integrals to volumetric and using arbitrariness of the selection of volume of V, we obtain

$$\frac{\partial}{\partial t}\left[em\rho + e_1\rho_1(1-m)\right] + \nabla(e\vec{u}\rho - \lambda\nabla T) + \nabla(p\vec{u}) = 0. \quad \text{(IX.1.5)}$$

Page 241.

3. If is examined the motion of homogeneous liquid, then the equation of energy (IX.1.5) can be simplified. Using the fact that regarding $\varepsilon + p/\rho = i$ and on the strength of the equation of the continuity of $\nabla(\vec{u}\rho) = -\partial\,(m\rho)/\partial t,$ we have

$$m\rho \frac{\partial}{\partial t} [i+\varepsilon_1\rho_1(1-m)] + \rho\vec{u}\,\nabla i - \frac{\partial}{\partial t}(mp) = \nabla(\lambda\,\nabla T).$$

It is convenient to express in this equation enthalpy i by temperature and pressure. Then

$$di = C_p(dT - \delta\,dp)$$

($C_p$ - heat capacity at constant pressure; δ is the Joule coefficient of - Thomson). Hence we will obtain

$$m\rho C_p \frac{\partial T}{\partial t} + C_1 \frac{\partial T}{\partial t} + \rho\vec{u} C_p \nabla T =$$
$$= \frac{\partial (mp)}{\partial t} + m\rho C_p \delta \frac{\partial p}{\partial t} + \rho\vec{u} C_p \delta \nabla p + \nabla(\lambda\,\nabla T). \qquad (IX.1.6)$$

In those regions where the motion of liquid is absent, which equation transfer/converts to the usual equation of thermal

conductivity

$$(m\rho C_p + C_1)\frac{\partial T}{\partial t} = \nabla(\lambda\nabla T).$$

Filtration motion is stabilized considerably faster than thermal field. Therefore appears the problem, in which one should consider only the transiency of thermal field, the problem of heat convection during the stationary filtration:

$$(m\rho C_p + C_1)\frac{\partial T}{\partial t} + \rho C_p\vec{u}\,\nabla T = \rho\vec{u}C_p\delta\,\nabla p + \nabla(\lambda\nabla T). \qquad \text{(IX.1.7)}$$

Comparing the second terms of the right and left sides of the equation (IX.1.7), it is easy to ascertain that already at comparatively small rates of filtration thermal conductivity in the direction of the motion of liquid can be disregarded everywhere, besides the ranges of an abrupt change in the temperature. It is really/actually:

$$\frac{|\nabla(\lambda\nabla T)|}{|\rho C_p\vec{u}\,\nabla T|} \approx \frac{\lambda}{\rho C_p u L} \approx \frac{\alpha\mu}{k\,\Delta p}$$

(L is a significant dimension; $\Delta p$ - the pressure differential at a distance L; $\alpha$ is a thermal diffusivity). With $\alpha \approx 10^{-3}$ $cm^2/s$; $\mu$ = 10 cp = 0.1 poise; $k = 10^{-9}$ $cm^2$ = 0.1 d; $\Delta p$ = 10 $kgf/cm^2$ = $10^7$ $dyn/cm^2$ we have

$$\frac{\alpha\mu}{k\,\Delta p} \approx 0{,}01.$$

From the given estimation it follows that the thermal conductivity causes only the local redistribution of temperature, and heat transfer up to large distances is connected with convection.

Page 242.

On the strength of this fact thermal conductivity in the direction of filtration flow usually they disregard, taking into account only thermal conductivity in the direction, perpendicular to motion.

4. Let us examine now the problem of the launching/starting of gas well with constant output, by taking into account the appearing in this case changes in the temperature. The basic system of equations can be, neglecting thermal conductivity, represented in the form:

$$\frac{\rho k}{\mu}\frac{\partial p}{\partial r}=\frac{1}{r}q; \quad q(r,\ t)=u(r,\ t)\rho r;$$

$$\frac{\partial(m\rho)}{\partial t}=-\frac{1}{r}\frac{\partial q}{\partial r};$$

$$C\frac{\partial T}{\partial t}-C_p\frac{q}{r}\frac{\partial T}{\partial r}-m(\rho C_p\delta+1)\frac{\partial p}{\partial t}+C_p\delta\frac{q}{r}\frac{\partial p}{\partial r}=0. \quad \text{(IX.1.8)}$$

Values $\rho$, $\delta$ and $\mu$ are the known functions of pressure and temperature, in particular

$$\rho = \frac{p}{z(p, T)RT}. \qquad \text{(IX.1.9)}$$

Let us place $\Theta = T/T_0,\ p^0 = p/p_0,$ where $p_0$ and $T_0$ - the initial pressures and temperature, whereupon the system (IX.1.8) will be led to the form:

$$\frac{p^0}{\mu^0 z\Theta}\frac{\partial p^0}{\partial r} = \frac{Q}{r}; \quad \frac{\partial}{\partial t}\left(\frac{p^0}{z\Theta}\right) = \frac{\varkappa}{r}\frac{\partial Q}{\partial r};$$

$$\left(1 + \frac{\beta C^0 p^0}{z\Theta}\right)\frac{\partial\Theta}{\partial t} - \varkappa\beta C^0 \frac{Q}{r}\frac{\partial\Theta}{\partial r} - \beta\frac{\partial p^0}{\partial t} -$$

$$-\frac{\beta C^0 D p^0}{z\Theta}\frac{\partial p^0}{\partial t} + \varkappa\beta C^0 D\frac{Q}{r}\frac{\partial p^0}{\partial r} = 0. \qquad \text{(IX.1.10)}$$

Here

$$\mu^0 = \frac{\mu}{\mu_0}; \quad Q = \frac{q\mu_0 R T_0}{k p_0^2}; \quad \varkappa = \frac{k p_0}{\mu_0 m}; \qquad \text{(IX.1.11)}$$
$$C^0 = \frac{C_p}{R}; \quad \beta = \frac{m p_0}{(1-m)\rho_1 C_1 T_0}; \quad D = \frac{\delta p_0}{T_0},$$

a subscript zero means that the corresponding values are undertaken with $p = p_0$, $T = T_0$.

System (IX.1.10) must be solved under the following supplementary conditions:

$$p^0(r, 0) = 1; \quad \Theta(r, 0) = 1; \quad Q(0, t) = Q_0. \qquad \text{(IX.1.12)}$$

It is easy to ascertain that the formulated problem is self-similar and has solution of the form:

$$p^0 = p^0(\xi); \quad \Theta = \Theta(\xi); \quad Q = Q(\xi); \quad \xi = {}^1/_2\, x/\sqrt{\varkappa t}, \quad \text{(IX.1.13)}$$

where $p^0$, $\Theta$ and $Q$ satisfy the system of the ordinary differential equations:

$$\begin{gathered}
-2\xi^2 \frac{d}{d\xi}\left(\frac{p^0}{z\Theta}\right) = \frac{dQ}{d\xi}; \\
\frac{\xi p^0}{\mu^0 z\Theta} \frac{dp^0}{d\xi} = Q; \\
\left[2\xi^2\left(1 + \frac{\beta C^0 p^0}{z\Theta}\right) + \beta C^0 DQ\right] \frac{d\Theta}{d\xi} = \\
= \left[2\beta\xi^2\left(1 + \frac{C^0 Dp^0}{z\Theta}\right) + \beta C^0 DQ\right] \frac{dp}{d\xi}; \\
p^0(\infty) = 1; \quad \Theta(\infty) = 1; \quad Q(+0) = Q_0.
\end{gathered} \quad \text{(IX.1.14)}$$

Page 243.

System (IX.1.14) can be integrated numerically.

Problem (IX.1.14) allow/assumes also the sufficiently simple approximate solution, based on what the distortion of velocity fields under the effect of temperature changes is small. Actually, examining a change in the temperature of the particle of gas along its trajectory, it is possible to show that the temperature differs from its initial value for the value, which does not exceed the value of the integral Joule effect of - Thomson during a change of the pressure from the initial $p_0$ to the existing at the particular point layer of pressure p:

$$|\Delta T| \leqslant \left| \int_{p_0}^{p} \delta \, dp \right|. \qquad \text{(IX.1.15)}$$

For natural gases the Joule coefficient of - Thomson $\delta \approx$ 0.4-0.5° C/ (kgf/cm²). Therefore outside the direct vicinity of the gas well of change temperatures are very small. Near hole the distribution of the mass rates of filtration stabilized is the same as during steady motion. Therefore without special error it is possible to accept as for isothermal motion,

$$Q = Q_0 \exp(-\xi^2). \qquad \text{(IX.1.16)}$$

Let us assume now that the parameters of the problem make it possible to fit this value of $\xi_* \ll 1$, that

$$Q_0^2 \ln^2 \xi_* \ll 1 \qquad \text{(IX.1.17)}$$

and at the same time

$$\xi_*^2 \ll \beta C^0 Q_0. \qquad \text{(IX.1.18)}$$

The first of these inequalities provides the smallness of the

deviations of pressures and, consequently, also the temperatures from their initial values; the second makes it possible to simplify the third equation (IX.1.14). Usual value $\beta C^{0}$ $10^{2}$. Therefore the system of inequalities (IX.1.17) and (IX.1.18) it is possible to easily satisfy already with $Q_0 \simeq 0.05$ (what is very large for a practice value).

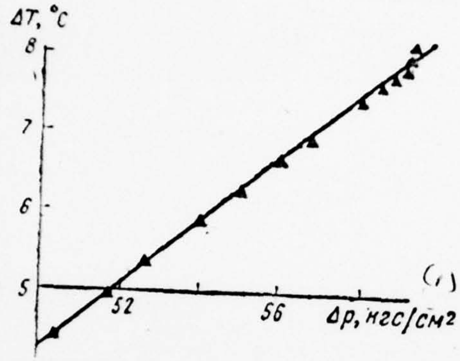


Fig. IX. 1

Key: (1) kgf/cm².

During the execution of inequality (IX.1.17) for $\xi > \xi^*$ we have

$$p^0(\xi) = 1 + {}^1/_2\, z_0 Q_0 \,\mathrm{Ei}\,(-\xi^2). \qquad \text{(IX.1.19)}$$

After this the distribution of temperature can be determined by the direct integration of the third equation of system (IX.1.14); then it takes the form (disregarding by low terms):

$$\Theta = 1 - D\,(1 - p^0) - \frac{\beta - D}{2}\, z_0 Q_0 \,\mathrm{Ei}\left(-\xi^2 - \frac{\beta C^0 Q_0}{2}\right). \quad \text{(IX.1.20)}$$

In range $\xi < \xi^*$ on the strength of inequality (IX.1.18) the last/latter equation (IX.1.14) is simplified and assumes the form:

$$\frac{d\Theta}{d\xi} = D\,\frac{dp^0}{d\xi}, \quad \text{(IX.1.21)}$$

whence

$$\Theta = \Theta_* + D\,(p^0 - p^0_*);$$
$$\Theta_* = \Theta\,(\xi_*); \quad p^0_* = p^0\,(\xi_*). \qquad \text{(IX.1.22)}$$

Substituting here expression (IX.1.20) and taking into account (IX.1.18), we obtain

$$\Theta = 1 - D\,(1 - p^0) - \frac{D-\beta}{2}\, z_0 Q_0\, \mathrm{Ei}\left(-\frac{\beta C^0 Q_0}{2}\right). \quad \text{(IX.1.23)}$$

This expression shows that in the examined self-similar solution of a change in the temperature at small $\xi$ (i.e. sufficiently long times) they repeat on the appropriate scale of pressure change.

Taking into account the dependence of temperature from pressure (IX.1.23) the density $\rho$ and ductility/toughness/viscosity $\mu$ gas in close to the hole of the range where the deviations of pressure from the initial value are great, prove to be known functions one only of

pressure. Therefore it is possible to determine pressure distribution in the range small $\xi$. Introducing the dimensionless function of Leybenzon

$$P^*(p^0) = 1 - \int_{p^0}^{1} \frac{p^0\,dp^0}{\mu^0 z\Theta(p^0)}, \qquad \text{(IX.1.24)}$$

we have for it approximately

$$P^*(p^0) = 1 + \frac{1}{2} Q_0 \operatorname{Ei}(-\xi^2); \quad \xi < \xi_*. \qquad \text{(IX.1.25)}$$

Page 245.

Systems dependence $P^*(p^0)$ on ln t, it is possible to define by usual method (in the same way as this was made in §4, chapter V) the

parameters of layer taking the correction into account for the nonisothermal character of motion.

The obtained above simple approximate solution will agree well with the results of the numerical integration of system (IX.1.14) which was carried out for several values of the parameters of the problem. The conclusion/derivation about linear communication/connection between changes in the pressure and temperature is confirmed by the experimental data [81], obtained on the holes of shebelinka gas field. As can be seen from Fig. IX.1, experimental points lie down well to the straight line, angular coefficient of which [0, 41° C/ (kgf/cm²) is close to the value of the Joule coefficient of - Thomson, calculated from the thermodynamic functions of gas. The results outlined here are obtained in V. M. Yentova's work [48].

§2. Elasto-plastic mode/conditions of oil stratum [20, 21].

1. During the derivation of the fundamental equations of the theory of elastic mode/conditions, we assumed that the strain of the skeleton of porous medium during a change of the pressure in layer is

elastic (i.e. the reversible during removal/taking load) and and what is more - linearly elastic. It would seem, for this there are all basis/bases, since pressure changes in the process of development of layer were small in comparison with the moduli of elasticity of the liquid to of the material of porous skeleton, and the very material of skeleton usually is the completely brittle body, which are deformed elastic up to destruction.

Nevertheless are determined indications [159] the fact that a change in the porosity of the rock/species, which compose oil stratum, with a change in the pressure of liquid bears inelastic character. This form of anelasticity is typical for the plastic state of material. The character of communication/connection between strains and stresses depends substantially on the direction of the process of deformation and even on its entire prehistory.

The typical diagram of the deformation of plastic material is shown in Fig. IX.2. Material undergoes loading, beginning from the undeformed state. If we, after achieving certain state, initiate to unload material, then instead of the return to the initial state will be obtained some new states. Discharging occurs in the manner that if the modulus of elasticity of material increased. With new load the

material is transformed on by unloading curve until it acquires before the not reached state.

Qualitatively similar picture is detected with the strain of the porous medium. Figure IX.3 shows the dependence of porosity on load for sand [159]; however, analogous dependence occurs also for other porous media. The reason for the fact that a change in the porosity bears "plastic" character, they are, it is natural, not the plastic deformations of separate grains of skeleton, but the irreversible changes in their relative location (repacking of grains).

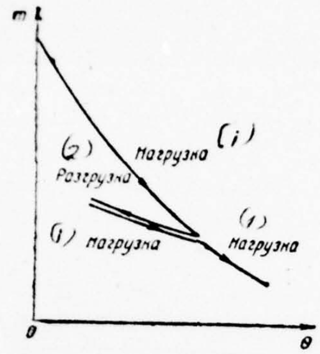


Fig. IX.2.

Key: (1). Load. (2). Discharging.

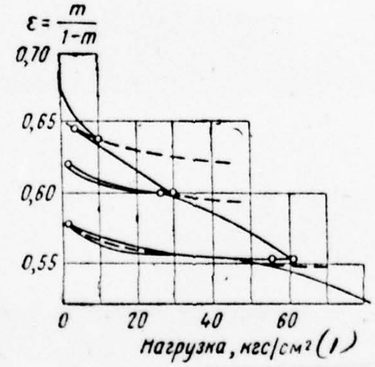


Fig. IX.3.

Key: (1). Load, kgf/cm².

The possibility of the repacking of particles of the porous medium qualitatively differs its behavior from the behavior of the material of the skeleton, which does not contain pores.

2. Discussing just as when deriving the equations of elastic mode/conditions, let us assume that the porosity is function one only of pressure and first invariant of the tensor of the fictitious stresses in the skeleton:

$$m = m(p, \Theta); \quad \Theta = \frac{1}{3}\sigma'_{ii}. \qquad \text{(IX.2.1)}$$

In accordance with this

$$\frac{\partial m}{\partial r} = \left(\frac{\partial m}{\partial p}\right)_\Theta \frac{\partial p}{\partial t} + \left(\frac{\partial m}{\partial \Theta}\right)_p \frac{\partial \Theta}{\partial r} \qquad \text{(IX.2.2)}$$

(derivative in terms of p is undertaken with constant θ, and derivative in terms of θ - with constant p).

As this was already shown when deriving the equations of elastic mode/conditions (see chapter II, §2), pressure change is compensated for by a change in the stresses in skeleton, so that

$$\frac{\partial(\Theta+p)}{\partial t_1}=0. \qquad \text{(IX.2.3)}$$

In other words, the sum of pressure and first invariant of the tensor of fictitious stresses is constant - it is balanced by the mass of the superincumbent rock/species, which virtually is not changed.

A change in the porosity of the medium with a change in the pressure of liquid with constant $\theta$ is caused exclusively by the elastic deformations of the material of skeleton. Therefore independent of the direction of deformation it is possible derived $(\partial m/\partial p)_{\Theta}=1/K_3$ to consider constant.

In contrast to this the dependence of porosity on the first invariant of the tensor of fictitious stresses $\theta$ is substantially irreversible.

Page 247.

Let us assume that it is described by the dependence, similar to the given in Fig. IX.2 dependence of m on stresses. Let us assume further that in layer occurs a single increase in the fictitious tensions, it is possible with their subsequent decrease. On the strength of equality (IX.2.3) this corresponds to the initial decompression with its subsequent partial increase. This diagram is of considerable interest in connection with the fact that during the first stage of the development of petrolium deposit usually occurs at first a decompression on deposit as a whole, then follows the period of the partial recovery of sheet pressure with the aid of the secondary methods - in essence of postcontour and inner-contour water-flood.

In accordance with that which was presented, let us assume that

the initial pressure in layer $p_0$ first monotonically descends: $\partial p/\partial t < 0$. In this case, $\partial\Theta/\partial t > 0$, and it is possible to count the derived $(\partial m/\partial\Theta)_p$ of constant and equal to certain value $1/K_1$. During a pressure increase of $(\partial\Theta/\partial t) < 0$, in derived $(\partial m/\partial\Theta)_p$ takes certain another value $1/K_2$. The discharging is "more rigid" - usually the module/modulus of discharging $K_2$ more the module/modulus of load $K_1$. The new loading of the section, already subjected to discharging, occurs with that value of module/modulus $K_1$.

Thus, for the characteristic of dependence of m on θ is sufficient to assign two value of the module/moduli: $K_1$ and $K_2$ and to indicate, to which from them one should use at each this torque/moment. This is easy to make, if we introduce the concept about the maximum during entire history of loading value θ.

Let us rely:

$$\Theta^*(x, t) = \max_{0 \leqslant \tau \leqslant t} \Theta(x, \tau).$$

Then, obviously, it is possible to write

$$\left(\frac{\partial m}{\partial \Theta}\right)_p = -\frac{1}{K_1} \text{ при } \frac{\partial \Theta^*}{\partial t} > 0;$$
$$\left(\frac{\partial m}{\partial \Theta}\right)_p = -\frac{1}{K_2} \text{ при } \frac{\partial \Theta^*}{\partial t} = 0. \qquad (IX.2.4)$$

If we pass to pressure, then taking into account (IX.2.3) it is possible to introduce function

$$p_*(x, t) = \min_{0 \leq \tau \leq t} p(x, \tau) \qquad (IX.2.5)$$

and to rewrite relationship/ratio (IX.2.4) in the form:

$$\left(\frac{\partial m}{\partial \Theta}\right)_p = -\frac{1}{K_1} \text{ при } \frac{\partial p_*}{\partial t} < 0;$$
$$\left(\frac{\partial m}{\partial \Theta}\right)_p = -\frac{1}{K_2} \text{ при } \frac{\partial p_*}{\partial t} = 0. \qquad (IX.2.6)$$

Now it is possible to extract the complete system of equations of the theory of the elastc-plastic mode/conditions of oil stratum.

Page 248.

Let us write the equation of the continuity:

$$\frac{\partial (m\rho)}{\partial t} + \operatorname{div} \rho\vec{u} = 0. \qquad \text{(IX.2.7)}$$

We will consider liquid elastic, and filtration - by the following to the law of darcys. Then, proceeding analogously with the derivation of the equations of elastic mode/conditions, let us arrive at the following system of relationship/ratios:

$$\frac{\partial p}{\partial t} = \varkappa \Delta p, \qquad \text{(IX.2.8)}$$

where

$$\begin{aligned}
\varkappa &= \varkappa_1 = \frac{kK_1^*}{\mu m} \quad \text{при} \quad \frac{\partial p_*}{\partial t} < 0;\\
\varkappa &= \varkappa_2 = \frac{kK_2^*}{\mu m} \quad \text{при} \quad \frac{\partial p_*}{\partial t} = 0;\\
\frac{1}{K_1^*} &= \frac{1}{K_{ж}} + \frac{1}{mK_1} + \frac{1}{mK_3};\\
\frac{1}{K_2^*} &= \frac{1}{K_{ж}} + \frac{1}{mK_2} + \frac{1}{mK_3};
\end{aligned} \qquad \text{(IX.2.9)}$$

$p_*(r, t)$ it is determined by relationship/ratio (IX.2.5).

In spite of resemblance to the equation of elastic mode/conditions, equation (IX.2.8) taking into account (IX.2.9) is nonlinear. The general methods of its solution do not exist. Virtually it is necessary to divide/mark off the range of motion on several zones, for part of which is correct the equation (IX.2.8) from $\varkappa = \varkappa_1$, and for the others - the same equation from $\varkappa = \varkappa_2$. For each zone the equation is linear. Therefore the nonlinearity of problem is exhibited only on the existence of the unknown band edges of the action of the different forms of equation (IX.2.8).

the formulation of the basic problems for the equations of elasto-plastic mode/conditions coincides in essence with the formulation of the problems for the equations of elastic mode/conditions. Certain special feature/peculiarity has only in the fact that during the laying out of the range of motion to zones is necessary additionally to indicate conditions for the "seam" of the solutions, obtained for different zones. These conditions make usual physical sense: the equality of pressures and fluid flows on both sides of band edge, whence we obtain

$$p_+(x,\ t) - p_-(x,\ t) = 0;\quad \frac{\partial p_+(x,\ t)}{\partial x} - \frac{\partial p_-(x,\ t)}{\partial x} = 0 \qquad \text{(IX.2.10)}$$

(is here taken into account, that the zones are distinguished only by the given bulk modulus).

3. Let us examine as an example the problem of the recovery of pressure and layer with the cessation of the operation of gallery in infinite layer.

Let the initial pressure in the layer, on boundary of which ($x$ = 0) is a drainage gallery, constantly is equal to P. Let, further, at moment $t$ = 0 pressure on gallery fall to certain value $p_0$ < P and remains constant for a period of time T, whereupon the selection of the liquid through gallery ceases, and pressure in layer it begins to be restored.

Page 249.

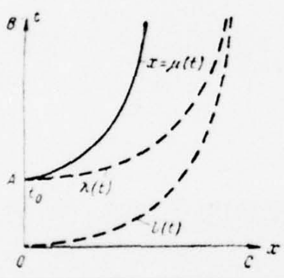


Fig. XI.4.

In the one-dimensional motion in question the pressure satisfies equations

$$\frac{\partial p}{\partial t} = \varkappa_1 \frac{\partial^2 p}{\partial x^2} \quad \left(\frac{\partial p}{\partial t} \leqslant 0\right); \qquad \frac{\partial p}{\partial t} = \varkappa_2 \frac{\partial^2 p}{\partial x^2} \quad \left(\frac{\partial p}{\partial t} > 0\right). \quad \text{(IX.2.11)}$$

under the conditions

$$p(x, 0) = P; \quad p(0, t) = p_0 \quad (0 < t < T);$$
$$\frac{\partial p(0, t)}{\partial x} = 0 \quad (t > T). \quad \text{(IX.2.12)}$$

• It is obvious that the plane alternating/variable x, t is divide/marked off into two range by line $x = \mu(t)$, on which

$$\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial x^2} = 0.$$

• This line, which depicts the wave front of discharging, begins in the point of the exchange of the boundary conditions $x = 0$, $t = T$

and is moved with an increase of t into the depth of layer. The position of this line must be determined in the course of the solution of problem.

We will search for the approximate solution of problem, utilizing a method of integral relationship/ratios (see Chapter V). In accordance with the common diagram of method, let us introduce the range of the effect of the initial change in the mode/conditions [its boundary let us designate by (t) ]. Unlike the making clear physical sense boundary μ (t) both these boundaries are conditional, that appear in connection with the application/use of an approximation method. Figure IX.4 they shows by dotted line.

At the initial stage (t < T) pressure in layer decreases, and the pressure distribution is determined by the first equation (IX.2.11). Accepting for pressure distribution approximation

$$p=\begin{cases} p_0+(P-p_0)\left(\frac{3x}{l}-\frac{3x^2}{l^2}+\frac{x^3}{l^3}\right) & (x\leqslant l) \\ P & (x\geqslant l), \end{cases} \qquad \text{(IX.2.13)}$$

for which first-order two derivatives becomes zero with $x=l$, and utilizing the first integral relationship/ratio (equation of material balance), let us find

$$l(t)=\sqrt{24\varkappa_1 t}. \qquad (IX.2.14)$$

Let us examine now strictly the reduction process of pressure (t > T).

Page 250.

Let us assume that the range of the effect of a change in the mode/conditions [0 < x < λ (t) ] will seize only the part of the disturbed range [0 < x < $l$ (t) ], so that during λ < x < $l$ pressure distribution retains the previous form, determined by relationship/ratios (IX.2.13) and (IX.2.14).

For pressure distribution in range 0 < x < λ (t) let us accept approximation

$$p = L + M\frac{x^2}{\lambda^2} + N\frac{x^3}{\lambda^3}, \qquad \text{(IX.2.15)}$$

satisfying boundary condition with x = 0, and coefficients L, M and N let us determine so that with x = λ expression (IX.2.15) continuously would be mated with expression (IX.2.13) with the preservation/retention/maintaining of the continuity of first-order two derivatives. The conditions of coupling give the system of three equations, solving which, we will obtain expressions for L, M and N as a function of the $\lambda/l$:

$$L = p_0 + (P - p_0)\frac{\lambda}{l};$$

$$M = 3(P - p_0)\frac{\lambda}{l}\left(1 - \frac{\lambda}{l}\right);$$

$$N = -(P - p_0)\frac{\lambda}{l}\left(1 - \frac{\lambda^2}{l^2}\right). \qquad \text{(IX.2.16)}$$

The position of the wave of discharging $\mu$ (t) is defined by that fact that during $x = \mu$ ($\tau$) pressure distribution, considered as function x, has a point of inflection, $\partial^2 p/\partial x^2 = 0$. Twice differentiating expression (IX.2.15) and utilizing formulas (IX.2.16), we obtain

$$\mu = -\frac{\lambda M}{3N} = \frac{\lambda}{1+\lambda/l}. \qquad \text{(IX.2.17)}$$

Thus, pressure distribution will be completely known, if we determine function λ (t). For determination λ (t) we will use the integral relationship/ratio, following from equations (IX.2.11). Integrating these equations - the first from μ (t) to λ (t), and the second from 0 to μ (t) and store/adding up the obtained relationship/ratios, we obtain

$$\frac{d}{dt}\int_0^{\lambda(t)} p(x,\ t)\,dx - p[\lambda(t),\ t]\frac{d\lambda}{dt} = (\varkappa_2-\varkappa_1)\frac{\partial p}{\partial x}\Big|_{x=\mu(t)} + \varkappa_1\left(\frac{\partial p}{\partial x}\right)_{x=\lambda(t)}. \tag{IX.2.18}$$

Let us require in order that distribution (IX.2.15) would satisfy integral relationship/ratio (IX.2.18). Utilizing the obtained above expressions for L, M, N and μ through λ and designating $\lambda/l = z$, z, we will obtain, by taking into account expression (IX.2.14) for $l(t)$, first-order equation for function z (t).

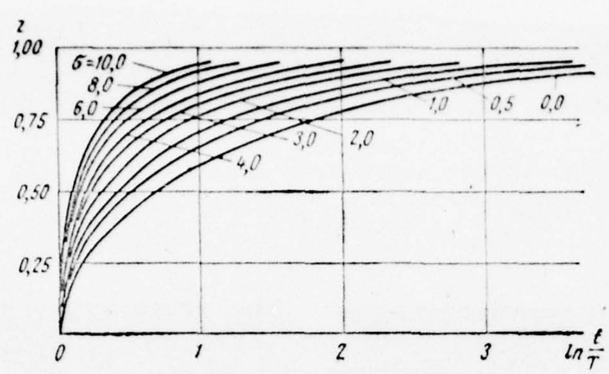


Fig. IX. 5.

This equation can be led to the form:

$$\frac{dt}{t}=\frac{4z\,(1+z)\,dz}{(1-z)\,[1+\sigma-z^2+z^2\,(1+z)\,(5-3z)]}, \qquad \text{(IX.2.19)}$$

where

$$\sigma=\frac{\varkappa_2-\varkappa_1}{\varkappa_1} \qquad \text{(IX.2.20)}$$

is the dimensionless relation, which characterizes the degree of the irreversibility of the deformations of the medium.

We have further

$$\lambda(T)=0; \quad z(T)=0. \qquad \text{(IX.2.21)}$$

Integrating equation (IX.2.19) taking this into account of condition, we obtain

$$\Phi(z, \sigma)=4\int_{0}^{z}\frac{\zeta(1+\zeta)\,d\zeta}{(1-\zeta)[1+\sigma-\zeta^{2}+\zeta^{2}(1+\zeta)(5-3\zeta)]}=\ln\frac{t}{T}. \qquad (IX.2.22)$$

The table of the values of the function of $\Phi(z, \sigma)$ is given in work [11]. Utilizing this table, is easy to find value of z, which corresponds this value of t (Fig. IX.5), and then to find pressure distribution, substituting in (IX.2.15) value of z and the corresponding to it values L, M and N, determined by formulas (IX.2.16).

From relationship/ratio (IX.2.22) it is clear that at all $T \leqslant t$;

t< - value $0 < z < 1$; with $t \to \infty\ z \to 1$, whence in accordance with (IX.2.17) it follows that $\mu/l \to {}^1/_2$. Thus, with large t the wave of discharging divides the disturbed range approximately half-and-half.

Let us note that in accordance with expressions (IX.2.15) and (IX.2.16)

$$z(t) = \frac{p(0,\ t) - p_0}{P - p_0}. \qquad \text{(IX.2.23)}$$

• Thus, the curve/graphs of Fig. IX.5 describe a pressure increase on gallery after its coverage.

Problems to p. 3.

1. Let p (x, t) are pressure distribution in semi-infinite layer, which satisfies during $0 < x < l(t)$ the second equation (IX.2.11), and with $l < x < \infty$ to the first, whereupon $p(x,\ t) \to P_0$ with $x \to \infty$.

Page 252.

To obtain integral relationship/ratio

$$\frac{1}{\varkappa_2}\frac{d}{dt}\int_0^l (p-P_0)\,dx+\frac{1}{\varkappa_1}\frac{d}{dt}\int_l^\infty (p-P_0)dx+$$
$$+\left(\frac{1}{\varkappa_1}-\frac{1}{\varkappa_2}\right)p(l,\ t)\,P_0\frac{dl}{dt}=-\left(\frac{\partial p}{\partial x}\right)_{x=0}. \qquad \text{(IX.2.24)}$$

To obtain also the subsequent integral relationship/ratios by analogy with the theory of elastic mode/conditions and the problems of the filtration of gas (see Chapter V).

Is which is the physical sense of relationship/ratio (IX.2.24)?

2. To achieve, utilizing a method integral relationship/ratios, the purpose of the pressure recovery near the hole, which worked in duration $0 < t < T$ with constant selection in the initially undisturbed layer, and then stopped.

The results outlined above were obtained in the works of G. I. Barenblatt and A. P. Krylov [20] and G. I. Barenblatt [11].

4. In the theory of elasto-plastic filtration significant turns out to be the question concerning self-similar solutions. It would seem, the solution of the problem of instantaneous source for the elasto-plastic mode/conditions of filtration must in accordance with §IV.5 be represented in the form:

$$P - p(x, t) = \frac{Q}{\sqrt{2\varkappa_1 t}} f(\xi); \quad \xi = \frac{x}{\sqrt{2\varkappa_1 t}}, \qquad \text{(IX.2.25)}$$

where f (ξ) it satisfies an equation

$$f''(\xi)+\xi f'(\xi)+f=0 \quad (0\leqslant\xi<\xi_0); \qquad \text{(IX.2.26)}$$
$$\alpha f''(\xi)+\xi f'(\xi)+f=0 \quad (\xi_0\leqslant\xi<\infty), \quad \alpha=\varkappa_1/\varkappa_2$$

and conditions

$$f'(0)=0, \quad \lim_{\xi\to\infty} f(\xi)=0, \qquad \text{(IX.2.27)}$$

whereupon $\mu(t)=\xi_0\sqrt{2\varkappa_1 t}$ - the coordinate of the wave of discharging.

In fact, the equations of the elasto-plastic mode/conditions of filtration contain not one new value with independent dimensionality ($\varkappa_1$ and $\varkappa_2$ have identical dimensionality). Therefore, set/assuming

$$\int_{-\infty}^{\infty}[P-p(x,\ 0)]dx=Q;\quad p(x,\ 0)\equiv 0\quad (x\neq 0)\quad \text{(IX.2.28)}$$

and utilizing usual dimensional considerations, we must obtain solution in the form (IX.2.25). It is easy to show, however that the solutions in the form (IX.2.25) does not exist. In fact, the solution to equations (IX.2.26) under the conditions (IX.2.27) takes the form, correspondingly, A exp $(-1/2\xi^2)$ $A_1$exp $(\xi^2/2\alpha)$ with $\xi < \lambda$ and $\xi > \lambda$. If we now write continuity conditions f and f' with $\xi = \lambda$, then with any $\lambda \neq 0$ for A and $A_1$ is obtained system of equations, incompatible/inconsistent with $\alpha \neq 1$.

Page 253.

This result is represented strange, because, would seem the solution of the problem of Cauchy after sufficient long time after start must "to forget" of the parts, connected with the initial condition, and to become self-simulating. Let us examine the solution

of the non-self-simulating problem of Cauchy [equation (IX.2.11)], that corresponds to the initial condition

$$P-p(x,\ 0)=(Q/l)\varphi(x/l), \qquad \text{(IX.2.29)}$$

where $l$ - certain parameter of the dimensionality of length, and function $\varphi(x/l)$ — is continuous, together with our derivative in terms of x, monotonically decreasing and besides such, that

$$\int_{-\infty}^{\infty}\varphi(\xi)\,d\xi=1. \qquad \text{(IX.2.30)}$$

On the strength of the theorem, demonstrated to S. L. Kamenomostskaya [51, 52], the solution of this problem of Cauchy exists uniquely. According to P-theorem it is represented in the form:

$$P-p(x,\ t)=\frac{Q}{\sqrt{\varkappa_1 t}}F\left(\frac{x}{\sqrt{2\varkappa_1 t}},\ \frac{l}{\sqrt{2\varkappa_1 t}},\ \frac{\varkappa_1}{\varkappa_2}\right). \qquad \text{(IX.2.31)}$$

Usual reasoning lies in the fact that with sufficiently small

$l$ the second argument of function F is unessential, whence is obtained representation (IX.2.25). The nontriviality of the position, which appears in the theory of the elasto-plastic mode/conditions of filtration, is explained by the fact that with $\eta = l(\varkappa_1 t)^{-1/2} \to 0$ the function of $F(\xi, \eta, \alpha)$ in the case $\alpha \neq 1$ does not approach the final limit, but it vanishes or infinity (depending on the value of $\alpha = \varkappa_1/\varkappa_2$) and besides so that there is this number $\beta$, that

$$\lim_{\eta \to 0} \frac{F(\xi, \eta, \alpha)}{\eta^{\beta}} = f(\xi, \alpha). \qquad (IX.2.32)$$

. Substituting this expression in (IX.2.31), we obtain, that the dominant term of the asymptotic behavior of the solution of the problem of Cauchy in $l \to 0$ (or, that the same, $t \to \infty$) takes the form:

$$P - p(x, t) = A(\varkappa_1 t)^{-1/2(1+\beta)} f(\xi, \alpha), \qquad (IX.2.33)$$

where $A = Ql^{\beta}$, that also suggests to search for the self-similar solution of the problem of Cauchy in the form (IX.2.33), whereupon

the parameter $\beta$ must be found in the course of the solution of problem. Substituting (IX.2.33) in (IX.2..11), we find for function f of the equation:

$$\begin{aligned} &\frac{d^2f}{d\xi^2}+\xi\frac{df}{d\xi}+(1+\beta)f=0 \quad (0\leqslant\xi\leqslant\xi_0);\\ &a\frac{d^2f}{d\xi^2}+\xi\frac{df}{d\xi}+(1+\beta)f=0 \quad (\xi_0\leqslant\xi<\infty). \end{aligned} \tag{IX.2.34}$$

Page 254.

From boundary conditions and the continuity conditions of pressure on the wave of discharging, we find

$$f'(0)=0;\quad f(\xi_0-0)=f(\xi_0+0), \qquad \text{(IX.2.35)}$$

whence and from the determination of the wave cf discharging

$$\frac{d^2f(\xi_0-0)}{d\xi^2}=\frac{d^2f(\xi_0+0)}{d\xi^2}=0$$

it is obtained continuity condition f' ($\xi$) cn the wave of discharging. From these conditions and the condition of the rapid decrease of function f on infinity, we find

$$\begin{aligned} f(\xi,\ \beta)&=Ae^{-\xi^2/2}\Phi\left(\frac{\beta}{2},\ \frac{1}{2},\ \frac{\xi^2}{2}\right) \quad (0\leqslant\xi\leqslant\xi_0);\\ f(\xi,\ \beta)&=Be^{-\alpha\xi^2/4}D_\beta(\sqrt{\alpha}\xi) \quad (\xi_0\leqslant\xi<\infty), \end{aligned} \qquad \text{(IX.2.36)}$$

(where $\Phi, D$ - the respectively degenerate hypergeometric function and

the function of hyperbolic cylinder, cm., [129]) and the system of two transcendental equations, which by single-valued form determines $\xi_0$ ($\alpha$) and $\beta$ ($\alpha$). It is natural that $\beta = 0$ with $\alpha = 1$. The obtained self-similar solution is interesting in that relation, that the index of the dimensionality of self-similar variable in it is not determined from dimensional considerations, but it is located from the condition for existence of the solution of problem as a whole. Such self-similar solutions are called the self-similar second-order solutions. The investigation of this problem was carried out in the work of G. I. Barenblatt and G. I. Sivashinskogo [23], in whom it is possible to find the parts of calculations and the results of numerical calculations.

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Pages 255-288.

Chapter X.

SPECIAL PROBLEMS IN [redacted] THE UNSTEADY FILTRATION OF INHOMOGENEOUS LIQUID.

§1. Displacement of interacting liquids.

In the present paragraph will be examined the consistent filtration of two completely mutually soluble liquids, forming during motion one phase. Of this type filtration currents are realized, for example, with the displacement of oil from layer by solvents, during the investigation of oil-bearing and aquifers with the aid of marked particles, and also in some processes of chemical technology.

1. Let us examine the process of the isothermal filtration of single-phase two-component mixture. The properties of this mixture (density and ductility/toughness/viscosity) are determined by two parameters - by mass concentration of one of the components c and by pressure p. During the displacement of the being mixed liquids by the chief characteristic of the filtration of single-phase two-component mixture is the complex mechanism of the transfer of mass, connected with the mixing of particles of both components as a result of a difference of the velocities at different points porous medium. The effect of mixing on heat transfer in the porous medium was noted already in §5 chapter IX. Due to the chaotic location of pore channels, the motion of liquid during filtration occurs along complex trajectories and at the distances of the order of the size/dimensions of pores the speed of each single particle of liquid can considerably

differ in value and direction from the average speed, equal to u/m (where u is rate of filtration, m - porosity). Therefore during motion in the porous medium of two-component mixture, the particles of each component are scattered relative to the initial position, in spite of the equality of the average speed for all particles. In this case, initial sharp boundary of two completely being mixed liquids turns out to be "that which was washed away". Erosion of boundary occurs, of course, and under the effect of molecular diffusion, but experiments show [153] that in the porous medium during filtration the mass transfer as a result of the deviation of speeds from the average values can occur many times (sometimes on several orders) is faster than the transfer, caused by molecular diffusion.

Page 256.

In order to consider the effect of mixing on the transfer of the component, which has concentration c, one should to the vector of convective transfer $\rho c\vec{u}$ add the additive term, connected with concentration change from one point to the next. The supplementary vector of the transfer of $\vec{q}_D$ can be written in the form

$$q_{Di} = A_{ij} \frac{\partial c}{\partial x_j}. \qquad (X.1.1)$$

The possibility of using a vector of the transfer in the indicated form is confirmed by processing data experiments in mixing in the porous medium. The tensor of $A_{ij}$ usually is called the tensor of dispersion, sometimes also by the tensor of convective diffusion. From symmetry, conditions it follows that in isotropic medium one of the principal axes of the tensor of $A_{ij}$ coincides with the direction of rate of filtration, and two others can be selected arbitrarily in the plane, perpendicular to vector $\vec{u}$. The components of the tensor $A_{ij}$ in the principal axes $K_1 = A_{11}$ and $K_2 = A_{22} = A_{33}$ are called respectively the coefficients of the longitudinal and transverse dispersion.

The coefficients $K_1$ and $K_2$ are determined by the mechanism of

mixing described above and therefore they are the functions of the average speed of motion (rate of filtration u) and of the properties of the porous medium ( , m, ...) and of the liquid ($\mu$, $\rho$, D):

$$K_i = f_i(u,\ D,\ \mu,\ \rho,\ l,\ m),\quad (i=1,\ 2), \qquad \text{(X.1.2)}$$

where D is a coefficient of molecular diffusion.

The dimensionality of the coefficients of dispersion $K_1$ and $K_2$ is $L^2/T$. Therefore

$$K_i = ul\varphi_i(\mathrm{Pe},\ \mathrm{N},\ \alpha_1,\ \alpha_2,\ \ldots), \qquad \text{(X.1.3)}$$

where $Pe = u\ell/D$ (Peclet's number); $N = D\rho/\mu$; $\alpha_1$, $\alpha_2$, ... they are the dimensionless parameters of the structure of pore space; $\varphi_i$ are a dimensionless function.

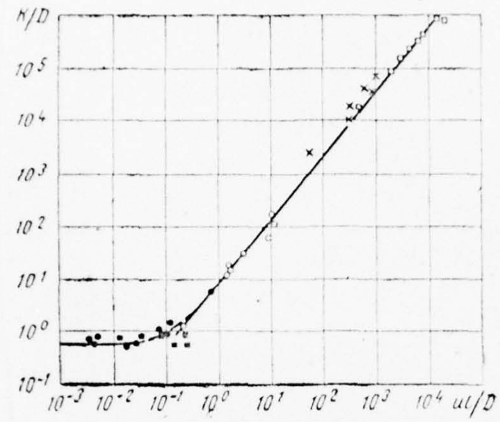
From formula (X.1.3) it follows that the dependence of the coefficients of dispersion on rate of filtration is exhibited through their dependence on Peclet's number. Figure X.1 gives the dependence of the dimensionless coefficient of the longitudinal dispersion K/D on Pe. This curve/graph is obtained as a result of processing the large number of experiments in the study of the distribution of the concentration of neutral impurity/admixture in homogeneous liquid

during filtration in uncemented sands [58]. As significant dimension , was accepted the mean grain diameter.

let us note that the experiments were carried out at the different values of parameter N, but of the noticeable effect this parameter on $K_1$ reveal/detected.

Fig. X.1.

[redacted]



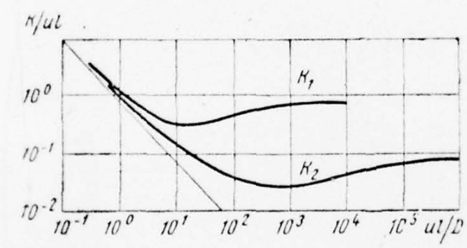




Fig. X.2.

In the region of the smallest rates of filtration when u << D, mechanical agitation is unessential in comparison with molecular diffusion and

$$K_1 = aD, \qquad (X.1.4)$$

where a is constant, which depends on the structure of pore space. In the range 1 < Pe < 100 is arrange/located the transition region, in which are significant both processes - molecular diffusion and mechanical agitation.

Finally, in region Pe > 100 effect of molecular diffusion on coefficients of dispersion is insignificant and $\Phi_1$ = const.

At numbers Pe > $10^5$ value $K_1/u$ begins to decrease with an increase Pe. In this range already pronounces the effect of inertial forces on the distribution of the rates in pores.

The dependence of the coefficient of the transverse dispersion $K_2$ on Peclet's number in principle is analogous to dependence $K_1$ (Pe). However, the effect of rate on $K_2$ begins to be exhibited with Peclet's considerably large numbers, than for $K_1$. Therefore with

Peclet's large numbers $K_1 >> K_2$ as this is evident from Fig. X.2, borrowed from the work of Marl and Potier [147]. The detailed analysis of the dependence of the tensor of dispersion on rate is brought in chapter II book A. Ban et al. [3], written by V. N. Nikolaevskiy, and also in Collins' book [58]. There there is a detailed bibliography.

Let us write now the equations of the consistent isothermal filtration of two mutually soluble incompressible fluids. The equations of the continuity of each component are derive/concluded in accuracy just as equation (VI.2.4), and they take the form:

$$\frac{\partial}{\partial t}(m\rho c_\lambda) + \operatorname{div}(\rho \vec{q}_\lambda) = 0 \quad (\lambda = 1, 2), \qquad (X.1.5)$$

where the $\vec{q}_\lambda$ the flow of this component, which can be expressed in the form:

$$q_{\lambda i} = c_\lambda u_i - A_{ij}\frac{\partial c_\lambda}{\partial x_j} = -\frac{kc_\lambda}{\mu}\frac{\partial p}{\partial x_i} - A_{ij}\frac{\partial c_\lambda}{\partial x_j}. \qquad (X.1.6)$$

. Page 258.

The substitution of expressions (X.1.6) in (X.1.5) reduces to the system of nonlinear equations for $c = c_1$ and p:

$$m\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0; \qquad (X.1.7)$$

$$m\frac{\partial c}{\partial t} + u_i\frac{\partial c}{\partial x_i} - \frac{\partial}{\partial x_i}\left(A_{ij}\frac{\partial c}{\partial x_j}\right) = 0. \qquad (X.1.8)$$

. With the study of fluid flows in the porcus medium with the aid of marked particles the displacing liquid (containing neutral impurity/admixture) it has the same physical properties, as displaced. Therefore system (X.1.7) - (X.1.8) is divide/marked off intc twc independent equations one of which determines velocity field, and the second serves for determining concentration. In this case, the second equation will be linear. In the majority of the problems, connected with the motion of marked particles, filtration can be considered establish/installed. Then equation (X.1.7) transfer/converts to the equation of Laplace.

During the investigation strictly cf the displacement of the being mixed liquids, for example the displacement of oil by solvents, the problem is simplified in connection with the fact that the rates of filtraticn in layer far from holes are small and change insignificantly. Thus, for instance, with $u$ = 350 m/yr $\approx 10^{-3}$ cm/s and $\ell$ = 0.01 cm. (which corresponds to permeability about $1\,\partial)$ $Pe \approx 1$ (since D usually of order - $10^{5}$ $cm^2/s$). Therefore the coefficients of dispersion can be considered not velocity-dependent, but at sufficiently low speeds (with $Pe < 10$) and equal between themselves (approximately equal to the coefficient of molecular diffusicn). Near holes current can be considered one-dimensional (radial); however, concentration distribution can and not be one-dimensional.

2. Let us examine some one-dimensicnal prcblems of the displacement of the being mixed liquids.

In the case of the one-dimensional rectilinear displacement of the incompressible fluids in the incompressible porous medium, equation (X.1.7) gives $u = u_0$ = const. If we dc not consider the

possible dependence of coefficient $K_1$ on the relation of ductility/toughness/viscosity, equation (X.1.8) will take the form:

$$m\frac{\partial c}{\partial t}+u_0\frac{\partial c}{\partial x}-K_1\frac{\partial^2 c}{\partial x^2}=0. \qquad \text{(X.1.9)}$$

. Since $u_0$ = const, also $K_1$ ($u_0$) = const.

Let liquid 1 be forced into the semi-infinite layer, initially filled by liquid 2. Then we have the following boundary and initial conditions: c (0, t) = 1, c ($\infty$, t) = 0, c (x, 0) = 0.

Page 259.

Applying the Laplace transform to equation (X.1.9), we have for an

$$c^* = \int_0^\infty c\,(x,\ t)\,e^{-\sigma t}\,dt$$

$$\sigma c^* + V\frac{dc^*}{dx} - K\frac{d^2c^*}{dx^2} = 0, \qquad (X.1.10)$$

where $V = u_0/m$; $K = K_1/m$.

Boundary conditions for c * will be written in the form:

$$c^*(0) = \frac{1}{\sigma}; \quad c^*(\infty) = 0.$$

The corresponding solution to equation (X.1.10) takes the form:

$$c^* = \frac{1}{\sigma}\exp\left\{-\frac{x}{\sqrt{K}}\left(\sqrt{a^2+\sigma} - a\right)\right\}, \qquad (X.1.11)$$

where the $a=\frac{iV}{2\sqrt{K}}$.

For the search of original on image (X.1.11) we utilize a known formula (see Chapter III)

$$\frac{1}{\sigma}\exp\left(-\frac{x}{\sqrt{K}}\sqrt{\sigma}\right)\leftrightarrow\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{x^2}{Kt}}\right). \qquad (X.1.12)$$

Furthermore, it is known that if the Laplace transform function $\phi$ (t) is F ($\sigma$), then the transformaticn of function exp ($-a^2t$) $\phi$ (t) will be F ($a^2 + \sigma$). By utilizing these

relationship/ratios, and also product rule for the Laplace transform, let us find after some transformations

$$c(x,\ t)=\frac{1}{2}\operatorname{erfc}\left(\frac{x-Vt}{2\sqrt{Kt}}\right)+\frac{1}{2}\exp\left(\frac{Vx}{K}\right)\operatorname{erfc}\left(\frac{x+Vt}{2\sqrt{Kt}}\right). \quad \text{(X.1.13)}$$

To the front of displacement, obvicusly, corresponds the vicinity of point x = Vt. With x, close to Vt, and with large t second term in formula (X.1.13) is negligibly small and

$$c\approx\frac{1}{2}\operatorname{erfc}\left(\frac{x-Vt}{2\sqrt{Kt}}\right). \quad \text{(X.1.14)}$$

Formula (X.1.14) represents the solution to equation (X.1.9),

that satisfies the following initial conditions: c (x, 0) = 1 with x < 0 and c (x, 0) = 0 with x > 0.

From formula (X.1.14) it follows that through sufficiently long time the transition zone, in which the concentration is changed from certain low value of $\varepsilon$ to $1-\varepsilon$, is expanded in the course of time proportionally $\sqrt{Kt}$. If we accept $\varepsilon = 0{,}005$, then the half of the width of transition zone is equal $4\sqrt{Kt}$.

Page 260.

For the case of radial displacement, equation (X.1.7) gives $u = q_0/r$, where $q_0$ = const. From equation (X.1.8) we will obtain

$$m\frac{\partial c}{\partial t}+\frac{q_0}{r}\frac{\partial c}{\partial r}-\frac{1}{r}\frac{\partial}{\partial r}\left(rK_1\frac{\partial c}{\partial r}\right)=0. \qquad \text{(X.1.15)}$$

. If molecular diffusion can be disregarded, then $K_1 = \lambda u$ or $K_1 = \lambda q_0$. Then instead of (X.1.15) we have

$$m\frac{\partial c}{\partial t}+\frac{q_0}{r}\frac{\partial c}{\partial r}-\frac{\lambda q_0}{r}\frac{\partial^2 c}{\partial r^2}=0. \qquad (X.1.16)$$

Equations (X.1.15) or (X.1.16) can be solved only numerically. For the approximate estimate of an increase in the transition zone, let us replace in equation (X.1.15) the variables $r^2 - 2q_0t = \zeta$, $t' = t$. Dependence $K_1$ (u) approximately let us accept in the form $K_1 = K_0 + \lambda u$. Then equation (X.1.15) is written as

$$\frac{\partial c}{\partial t}-4\frac{\partial}{\partial \zeta}\left\{\left[K_0(\zeta+2q_0t)+q_0\lambda\sqrt{\zeta+2q_0t}\right]\frac{\partial c}{\partial \zeta}\right\}. \qquad (X.1.17)$$

. If into the layer, which contains the only displacing liquid ($c = 0$), is pumped into liquid with concentration $c = 1$, then is the front of displacement (not allowing for dispersion) is moved according to the law $r=\sqrt{2q_0t}$. To evaluate concentration distribution near front during large t than r, i.e., $|\zeta| \ll 2q_0t$. Then expression in the brackets of signs the form: $2K_{010}t + \sqrt{2}(q_0t)^{3/2}\lambda$ can be removed as the sign of derivative in terms of $\zeta$. By set/assuming

$$\tau=4q_0(t^2-t_0^2)+\frac{8\sqrt{2}}{3}\frac{\lambda}{K_0}[(q_0t)^{3/2}-(q_0t_0)^{3/2}]$$

(where $t_0$ is certain constant), let us give equation (X.1.17) to the form:

$$\frac{\partial c}{\partial\tau}-K_0\frac{\partial^2c}{\partial\zeta^2}=0. \qquad (X.1.18)$$

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THE THEORY OF THE UNSTEADY FILTRATION OF LIQUID AND GAS (TEORIY--ETC(U)
JAN 77 G I BARENBLATT, V M YENTOV, V M RYZHIK

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In order to rate/estimate the rate of growth of transition zone, let us examine the self-similar solution to equation (X.1.18), that has the form, analogous (X.1.14):

$$c = \frac{1}{2}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{K_0\tau}}\right). \qquad (X.1.19)$$

This solution describes in variables $\zeta$, $\tau$ the propagation of the initially "stepped" distribution of concentration (i.e. so that

with $\tau = 0$ $c = 1$, if $\zeta < 0$, and $c = 0$, if $\xi > 0$). In alternating/variable r, t, these conditions will be expressed as follows: with $t = t_0$ $c = 1$, if $r^2 - 2q_0t_0 < 0$, and $c = 0$, if $r^2 - 2q_0t_0 > 0$.

Page 261.

In this case,, it goes without saying, remains valid the assumption that $|r^2 - 2q_0t_0| \ll 2q_0t_0$. By substituting in (X.1.19) instead of $\zeta$ and $\tau$ their expression through r and t, we will obtain

$$c=\frac{1}{2}\operatorname{erfc}\left\{\frac{r^2-2q_0t}{2\left[4q_0K_0(t^2-t_0^2)+\frac{8\sqrt{2}\lambda}{3}\left((q_0t)^{3/2}-(q_0t_0)^{3/2}\right)\right]^{1/2}}\right\}. \quad \text{(X.1.20)}$$

. To evaluate the width of transition zone, we utilize a condition of $\varepsilon = 0.005$, whence and $\frac{\varepsilon}{2\sqrt{K_0\tau}} < 2$, or

$$\frac{|r^2 - 2q_0t|}{2\left[4q_0K_0(t^2 - t_0^2) + \frac{8\sqrt{2}\lambda}{3}\left((q_0t)^{3/2} - (q_0t_0)^{3/2}\right)\right]^{1/2}} \leqslant 2. \qquad (X.1.21)$$

With the comparatively small times when the second term by brackets considerably exceeds the first (i.e. dispersion predominates above molecular diffusion), from condition (X.1.21), using also the fact that $/r^2 - wq_0t/ << 2q_0t$, is easy to establish/install for the width of transition zone the asymptotic expression

$$r - \sqrt{2q_0t} \approx 4\sqrt{\frac{2}{3}\lambda}(2q_0t)^{1/4} \approx 4\sqrt{\frac{2}{3}\lambda r}. \qquad (X.1.22)$$

For the long times when values r are such, that $K_0 >> q_0\lambda/r$,

$$r - \sqrt{2q_0t} \approx 2\sqrt{2K_0t} \approx 2\sqrt{\frac{K_0}{q_0}r}. \qquad (X.1.23)$$

As shows experimental check (Bentsen, Nielsen, [131], the formula, analogous (X.1.20), relatively describes well concentration distribution during radial displacement.

§2. Stability of the displacement of the nonmiscible and being mixed liquids from the porous medium.

During the study of the displacement of liquids in the porous medium, the basic problems of the stability of the obtained solutions. Physically the possibility of the emergence of instability is connected with the fact that during the penetration (because of random disturbances) of the particle of more labile liquid into the

region, occupied with less labile liquid, it proves to be under the action of high pressure gradients, than acted on it in the undisturbed state, and particle motion is accelerated. If the more labile liquid is displacing, this leads to the growth of disturbance/perturbations. As a result of this elementary approach (see I. A. Charnyy [119]) are obtained the same stability conditions, as when using a stricter theory.

The common/general/total method of the analysis of the stability of any system consists of the studies of its behavior after imposition on the ground state of slight disturbances.

Page 262.

1. Let us examine at first the simplest case - vertical motion at the constant velocity of the plane interface of two liquids, which have different densities and ductility/toughness/viscosities. This diagram is maximum both for the displacement by the being mixed and nonmiscible agents, if we disregard the width of transition zones.

The filtration of each of the liquids is described by the following equations (X-axis is directed vertically upward):

$$u_j = -\frac{k_j}{\mu_j}\left(\frac{\partial p}{\partial x} + \rho_j g\right); \quad v_j = -\frac{k_j}{\mu_j}\frac{\partial p}{\partial y}; \quad w_j = -\frac{k_j}{\mu_j}\frac{\partial p}{\partial z};$$
$$\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} + \frac{\partial w_j}{\partial z} = 0 \qquad (j = 1,\ 2). \qquad \text{(X.2.1)}$$

. The values, which relate to each of the phases, are here designated in index j = 1, 2; the permeability of $k_j$ are accepted different on both sides of interface in order to consider the possible incompleteness of the displacement of the nonmiscible liquids; in the latter case on both sides of boundary are different saturation and, therefore, are different relative permeability.

On interface, they must be fulfilled condition (see Chapter VI, §2) (VI.2.19) and (VI.2.21):

$$p_1 = p_2;$$
$$u_{n1} = u_{n2} = mV_n, \qquad \text{(X.2.2)}$$

where the $u_{nj}$ - the projection of rate of filtration on standard to interface; $V_n$ - the speed of boundary migration along the normal to it.

System (X.2.1) with conditions (X.2.2) has the following solution, which corresponds to the uniform displacement/movement of plane interface:

$$u_1 = u_2 = u_0; \quad v_1 = v_2 = 0; \quad w_1 = w_2 = 0;$$
$$p_1 = p_1^{(0)} = p_0 - \left(\frac{\mu_1}{k_1} u_0 + \rho_1 g\right)(x - Vt) \qquad (x - Vt < 0);$$
$$p_2 = p_2^{(0)} = p_0 - \left(\frac{\mu_2}{k_2} u_0 + \rho_2 g\right)(x - Vt) \qquad (x - Vt > 0). \quad (X.2.3)$$

In this case, $V_n = V = \frac{u_0}{m}$. The equation of the undisturbed interface takes form $x = Vt$.

Let us examine the solution of system (X.2.1), that differs from (X.2.3) in terms of slight disturbances. For this, let us place

$$u_j = u_0 + \varepsilon u_j^*; \quad v_j = \varepsilon v_j^*; \quad w_j = \varepsilon w_j^*; \quad p_1 = p_1^{(0)} + \varepsilon p_1^*; \quad p_2 = p_2^{(0)} + \varepsilon p_2^*, \tag{X.2.4}$$

where the $\varepsilon$ - low value.

The equation of interface takes the form:

$$x = x_0(y,\ z,\ t) = Vt + \varepsilon x^*(y,\ z,\ t). \qquad \text{(X.2.5)}$$

. Page 263.

For the disturbed motion we have a system of equations

$$u_j^* = -\frac{k_j}{\mu_j}\frac{\partial p_j^*}{\partial x};\quad v_j^* = -\frac{k_j}{\mu_j}\frac{\partial p_j^*}{\partial y};\quad w_j = -\frac{k_j}{\mu_j}\frac{\partial p_j^*}{\partial z};$$
$$\frac{\partial u_j^*}{\partial x} + \frac{\partial v_j^*}{\partial y} + \frac{\partial w_j^*}{\partial z} = 0. \qquad \text{(X.2.6)}$$

By using the smallness of the distortion of boundary, can be attributed conditions (X.2.2) to the undisturbed interface x = Vt. Then with an accuracy to small the order of the aaaa of boundary conditions takes the form himself:

$$u_1^* = u_2^* = m\,\frac{\partial x^*}{\partial t}$$

(1) при $x = Vt$;

$$p_1^* - p_2^* = \left[\left(\frac{\mu_1}{k_1} - \frac{\mu_2}{k_2}\right) u_0 + (\rho_1 - \rho_2)\, g\right] x^*. \qquad \text{(X.2.7)}$$

(1) with

To conditions (X.2.7) one should connect also the conditions of fading all disturbance/perturbations with $x \to \pm \infty$, since it is assumed that the disturbance/perturbation appears near interface.

Random disturbance can be decomposed into Fourier integral in terms of y and z. Therefore for stability analysis, it suffices to examine the development of **elementary** sinusoidal disturbance. For this, let us present x * **and** $p_j^*$ **in** the form of products

$$\begin{aligned} x^* &= X(t)\exp(i\gamma_1 y + i\gamma_2 z) \qquad (i = \sqrt{-1}); \\ p_1^* &= P_j(x,\ t)\exp(i\gamma_1 y + i\gamma_2 z) \quad (j = 1,\ 2), \end{aligned} \tag{X.2.8}$$

where X (t) and $P_j(x, t)$ - the amplitude cf disturbance/perturbations x * and $p_j^*$.

By substituting expressions (X.2.8) in equation (X.2.6), we will cbtain that the function of $P_j(x, t)$ must take the form:

$$P_j(x, t) = P_j^{(1)}(t)\exp(\gamma x') + P_j^{(2)}(t)\exp(-\gamma x') \qquad (\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}); \quad (x' = x - Vt). \tag{X.2.9}$$

The conditions of fading disturbance/perturbations at infinity give:

$$P_1 = P^{(1)}(t)\exp(\gamma x'); \quad P_2 = P^{(2)}(t)\exp(-\gamma x'). \qquad (X.2.10)$$

. Substituting these expressions in conditions (X.2.7) and eliminating $P^{(1)}$ and $P^{(2)}$, we obtain the following equation:

$$\frac{dX}{dt} = -\frac{N\gamma X}{m\left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2}\right)}; \quad N = \left(\frac{\mu_1}{k_1} - \frac{\mu_2}{k_2}\right)u_0 + (\rho_1 - \rho_2)g. \qquad (X.2.11)$$

. From equation (X.2.11) it follows that the $X = X_0 \exp\left(-\frac{N\gamma t}{m\left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2}\right)}\right)$, where $X_0$ is the initial amplitude of disturbance/perturbation. Therefore, if

$$N = \left(\frac{\mu_1}{k_1} - \frac{\mu_2}{k_2}\right)u_0 + (\rho_1 - \rho_2)g > 0, \qquad (X.2.12)$$

that the initial disturbances attenuate in the course of time, in the contrary case - grow/rise.

Page 264.

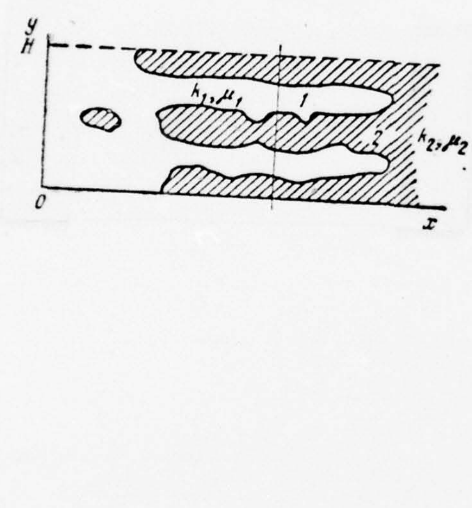


Fig. X.3.

Since in condition (X.2.12) does not enter the parameter the disturbance/perturbations $\gamma$, this condition are correct for the disturbance/perturbations of arbitrary form. Is concealed by form, the boundary migration of section stable, when is satisfied condition (X.2.12). If condition (X.2.12) is not satisfied, interface becomes unstable and is divide/marked off into the separate "languages" of complex and random form (Fig. X.3). When the action of the force is unessential, inequality (X.2.12) means that the interface is stable when the displacing liquid possesses larger viscosity than displaced, and is unstable otherwise. The action of the force contributes to the stability of boundary, if displacement goes from bottom to top displacing liquid it possesses larger density.

In the case of the nonmiscible liquids, the examined interface is actually the end position of the jump of the saturation when saturation on both sides of jump are different, but are constant. In the general case the saturation of the displacing phase in front of front and the saturation of the displaced phase behind front are not equal to zero. From each side of boundary are different not only the viscosity, but also permeability of each of the phases. Therefore in the case of the nonmiscible liquids, the stability is determined not by the relationship/ratio of viscosity, but by the relationship/ratio of mobilities, i.e., the values of $k_i/\mu_i$. The relation of

mobilities can be designed on the curves of relative permeability, on the strength of saturation on both sides of jump, which in turn, are determined from formulas (VI.2.29) and depend on the relation of viscosity. With an increase of the relation of viscosity $M =, \mu_2/\mu_1$ the relation of mobilities $M * = k_1\mu_2/k_2\mu_1$ also increases, but $M *$ becomes more than unity, i.e., stability is disrupted only when M considerably it exceeds unity. With the usual form of the curves of a relative permeability of the type of those depicted on Fig. VI.5 the relation of viscosity by which begins the instability, it is approximately 10-15. The Typical dependence of M * on M is shown in Fig. X.4.

The experiments in the displacement of oil by water, carried out B. Ye. Kisilenko [55] on the transparent models of layer with the filled porous medium, they showed that the instability begins in the relation of the viscosity of phases of approximately 12-13.

Page 265.

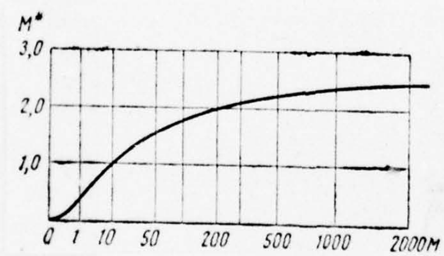


Fig. X.4.

If plane interface is unstable, then in the course of time it is divide/marked off into the large number of separate "languages" of irregular form (see Fig. X.3).

If because of the heterogeneity of flow saturation at certain point changes by low value, then this disturbance/perturbation is spread, without attenuating and without growing. Actually, the equation of continuity during plane two-phase flow they take the form (see Chapter VI):

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + m\frac{\partial s}{\partial t}; \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \qquad \text{(X.2.13)}$$
$$u = u_1 + u_2; \quad v = v_1 + v_2.$$

Furthermore, on the strength of the generalized law of Darcy (VI.2.1), it is possible to write

$$u_1 = uF(s); \quad v_1 = vF(s);$$

$$F(s) = \frac{f_1(s)}{f_1(s) + \frac{1}{M} f_2(s)}. \qquad (X.2.14)$$

. Then instead of the first of the equations (X.2.13) we have

$$m \frac{\partial s}{\partial t} + uF'(s) \frac{\partial s}{\partial x} + vF'(s) \frac{\partial s}{\partial y} = 0. \qquad (X.2.15)$$

. For undisturbed flow $u = u_0$, $s = s_0$, whereupon the velocity of propagation of jump, according to the condition of Bakley-Leverett (VI.2.30), is different $V = u_0/m\, F'(s_0)$. By introducing the values of the velocity disturbance and saturation, we will obtain from

(X.2.15)

(1)or

$$\frac{u_0}{m} F'(s_0) \frac{\partial s^*}{\partial x} + \frac{\partial s^*}{\partial t} \text{ (1) или } \frac{\partial s^*}{\partial t} + V \frac{\partial s^*}{\partial x} = 0. \qquad \text{(X.2.16)}$$

From this equation it is evident that the disturbance/perturbations of saturation near jump are spread, without attenuating, at a velocity, equal to the velocity of jump. Thus, the account of the disturbance/perturbations of saturation does not lead to a change in the stability condition.

2. Above was examined stability sharp the phase boundary, on which act the only forces of gravity and liquid resistance. However, during the displacement of the nonmiscible liquids to stability, can affect surface forces, and in the case of the being mixed displacement - dispersion and molecular diffusion. The leveling action of capillary forces on the porous medium in the general case connected with the processes of the redistribution of saturation, and for stability analysis it is necessary to examine the disturbance/perturbations of current in transition zone.

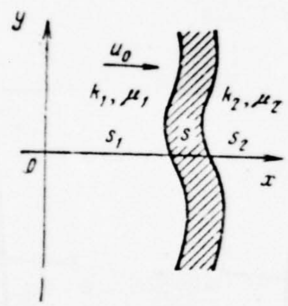


Fig. X.5.

We will be restricted to conditions, when the wavelength of disturbance/perturbation is great in comparison with the width of transition zone in order that the action of capillary forces it was possible to consider only under boundary conditions. In this case for the study of outside narrow transition zone it is possible to utilize equations (X.2.1). This approach, as we will see further, is connected with sufficiently essential limitations. For the simplicity of lining/calculations, we will be restricted to the case of flat/plane horizontal filtration. On the basis of formulas (VI.2.1) chapter VI, the equation of two-phase filtration in transition zone, let us write in the form:

$$u_j = -\frac{k}{\mu_j} f_j(s) \frac{\partial p_j}{\partial x}; \quad v_j = -\frac{k}{\mu_j} f_j(s) \frac{\partial p_j}{\partial y} \quad (j = 1,\ 2). \qquad \text{(X.2.17)}$$

$$w_j = 0.$$

Let us assume that a pressure difference in phases is equal to

capillary pressure - known function of saturation (see Chapter VI, §3):

$$p_2 - p_1 = p_c(s). \qquad (X.2.18)$$

The equations of continuity have usual form (X.2.13).

By combining equations (X.2.17) and (X.2.13), it is possible to obtain system for $u = u_1 + u_2$, $v = v_1 + v_2$ and $s$:

$$m\frac{\partial s}{\partial t} + uF'(s)\frac{\partial s}{\partial x} + vF'(s)\frac{\partial s}{\partial y} - a^2 m\left(\frac{\partial^2 \Phi(s)}{\partial x^2} + \frac{\partial^2 \Phi(s)}{\partial y^2}\right) = 0;$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0;$$

$$\frac{\partial}{\partial x}\left(\frac{v}{\varphi(s)}\right) = \frac{\partial}{\partial y}\left(\frac{u}{\varphi(s)}\right) \qquad (X.2.19)$$

(designation see chapter VI, §4); $\phi$ (s) = $f_1$ (s) + 1/M $f_2$ (s).

In order to obtain boundary conditions for disturbance/perturbations, let us integrate equations (X.2.19) for x along transition zone, by considering that the interface slightly bent (Fig. X.5). In this case, let us disregard the terms of the order of the width of zone and the squares of derivatives in terms of y. Then instead of the first equation (X.2.19) we obtain

$$m(s_1-s_2)\frac{\partial x_0}{\partial t}+u^{(2)}F(s_2)-u^{(1)}F(s_1)+$$
$$+[v^{(1)}F(s_1)-v^2F(s_2)]\frac{\partial x_0}{\partial y}-a^2m(\Phi_1-\Phi_2)\frac{\partial^2 x_0}{\partial y^2}=0. \qquad (X.2.20)$$

. Page 267.

Here index 2 designated values to the right transition zone, and by index 1 - to the left of it. Integration of other two equations (X.2.19) gives

$$u^{(1)}=u^{(2)};$$
$$\frac{v^{(1)}}{\varphi_1}-\frac{v^{(2)}}{\varphi_2}=-u^{(2)}\left(\frac{1}{\varphi_1}-\frac{1}{\varphi_2}\right)\frac{\partial x_0}{\partial y}. \qquad (X.2.21)$$

Transfer/converting to disturbance/perturbations, let us accept, as earlier,

$$x_0 = Vt + \varepsilon x^*;$$
$$u_f = u_0 + \varepsilon u_f^*; \quad v_f = \varepsilon v_f^*.$$

Then for disturbance/perturbations we obtain from (X.2.20)

$$\frac{\partial x^*}{\partial t} - \frac{u^*}{u_0} V - a^2 \frac{\Phi_1 - \Phi_2}{s_1 - s_2} \frac{\partial^2 x^*}{\partial y^2} = 0 \qquad \text{(X.2.22)}$$

and from (X.2.21)

$$u_1^* = u_2^* = u^*; \quad \frac{v_2^*}{\varphi_1} - \frac{v_2^*}{\varphi_2} = -u_0\left(\frac{1}{\varphi_1} - \frac{1}{\varphi_2}\right)\frac{\partial x^*}{\partial y}. \qquad (X.2.23)$$

The second of the equations (X.2.23), obviously equivalent to the latter from equations (X.2.7) when $\rho_1 = \rho_2$ can be rewritten in the form:

$$p_1^* - p_2^* = u_0\left(\frac{\mu_1}{k_1} - \frac{\mu_2}{k_2}\right)x^*; \quad k_j = k\varphi_j. \qquad (X.2.24)$$

Condition (X.2.22) replaces the second equality (X.2.7). Since, as earlier, it is assumed that on both sides the boundaries current potential, $\rho_j^*$ and x * again can be expressed by formulas (X.2.8) - (X.2.10).

By utilizing conditions (X.2.7) and (X.2.22) and by eliminating P 1 (t) and P 2 (t), we will obtain for X (t) equation

$$\frac{dX}{dt} = -\gamma\left[\frac{1-M^*}{1+M^*}V + a_1^2\gamma\right] \qquad a_1^2 = \frac{\Phi_1-\Phi_2}{s_1-s_2}a^2; \quad M^* = \frac{k_1\mu_2}{k_2\mu_1}. \tag{X.2.25}$$

Consequently, the stability of boundary is determined by condition

$$\frac{1-M^*}{1+M^*}V + a_1^2\gamma > 0. \qquad \text{(X.2.26)}$$

Thus, the interface of the nonmiscible liquids in the porous medium can become stable under the action of capillary forces even if $1 - M^* < 0$, i.e., condition (X.2.12) is not satisfied. Let $M^* > 1$, i.e., not allowing for capillary forces the boundary be unstable. Then condition (X.2.26) all the same is satisfied, if $\gamma > \frac{M^*-1}{M^*+1}\frac{V}{a_1^2}$. This means that if the wavelength of the disturbance/perturbation of boundary $\lambda = 2\pi/\gamma$ is shorter than the critical value of $\lambda_c$, then boundary remains stable. The critical wavelength of $\lambda_c$ is determined by formula

$$\lambda_c = \frac{2\pi}{\gamma_c} = \frac{2\pi a^2}{V}\frac{\Phi_1 - \Phi_2}{s_1 - s_2}\frac{M^*+1}{M^*-1}. \qquad (X.2.27)$$

. Page 268.

In the work of Chuok, etc. [135] was investigated the stability of the interface of liquids in the flat/plane slotted model where the action of surface forces with bending of boundary leads to the

emergence of a pressure difference on both sides of it. This pressure difference is directed is concealed by form, which contributes to the equalization of boundary.

We propose to the reader independently to investigate the stability of interface in this case, after using Laplace's formula for a pressure difference

$$p_c = \alpha\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \approx \alpha\left(-\varepsilon\frac{\partial^2 x^*}{\partial y^2}+\frac{1}{h}\right), \qquad \text{(X.2.28)}$$

where $\alpha$ - surface tension; h is a half of the width of slot.

In this case the critical wavelength of disturbance/perturbation takes the form:

$$\lambda_c = 2\pi\sqrt{\frac{\alpha}{(\mu_2-\mu_1)\frac{u_0}{k}-(\rho_1-\rho_2)g}}. \qquad \text{(X.2.29)}$$

The experiments of Chuok [135] and of B. Ye. Kisilenko [55] confirm that the interface remains stable even with the unfavorable relationship/ratio of mobilities, if the velocity of displacement is sufficiently small because of this the width of model shorter than the critical wavelength of disturbance/perturbation.

3. The derivation of formulas (X.2.26) and (X.2.27) was made on the assumption that the width of transition zone is considerably shorter than the wavelength of disturbance/perturbation. In accordance with results §3 chapter VI the width of transition zone is proportional $a^2/V$. Therefore the assumption pointed out above is correct only with M *, close to unity, and high value $(M^* - 1)^{-1}$. There is a considerable range of sizes M, with which M * it is close

to unity (see Fig. X.4).

Nevertheless formulas (X.2.26) and (X.2.27) are accurate only for a "weak" instability, i.e., for M *, close to unity, and they are in this sense asymptotic. In order to investigate stability in all range of a change in the relation of viscosity, one should utilize a complete system of equations (X.2.26). This problem thus far to end/lead is not solved.

The critical wavelength of the disturbance/perturbation of the $\lambda_c$, which divides the regions of stable and unstable displacement, in the general case is the function of parameters $a^2$, V and M = $\mu_2/\mu_1$. From dimensional considerations then it follows

. Page 269.

$$\lambda_0 = \frac{a^2}{V}\psi(M). \qquad (X.2.30)$$

If M it is close to that value of the $M_e$, with which M * becomes equal to unity, then expression (X.2.30) must approach a dependence (X.2.27).

Let us examine separately the stability of interface with respect to the one-dimensional disturbance/perturbations of saturation upon consideration of capillary forces, i.e., investigate the stability of the stabilized zone. The distribution of saturation in the stabilized zone is the solution to the equation of rapoport - Lis (VI.3.4) form $s = \phi_0 (x - Vt) = \phi_0 (\bar{x})$ and is expressed by formula (VI.3.12). Let us pass in the equation of rapoport - Lis, written in the form (VI.4.34), instead of x and t to the new independent variable $\bar{x}_0$ (s) and t and of the new unknown function $x - Vt = \bar{x}$; then this equation will take the form:

$$-\frac{\partial\bar{x}}{\partial t} + \frac{u}{m}F'(s) - a^2\frac{\partial}{\partial x_0}\left(\frac{\Phi'(s)\dfrac{\partial\bar{x}}{\partial x_0}}{\dfrac{\partial s}{\partial x_0}}\right) = 0. \qquad (X.2.31)$$

Let us introduce now the slight disturbances of the position cf point with saturation s in the stabilized zone of $\bar{x} = \bar{x}_0(s) + \varepsilon\bar{x}^*$.

for $\bar{x}^*$ we have equation

$$-\frac{\partial \bar{x}^*}{\partial t} + \left[-V + \frac{u_0}{\cdot m} F_1(\bar{x})\right] \frac{\partial \bar{x}^*}{\partial x} + a^2 F_2(\bar{x}) \frac{\partial^2 \bar{x}^*}{\partial \bar{x}^2} = 0, \qquad \text{(X.2.32)}$$

where $F_1$ (x) = F' (s); $F_1(\bar{x}) = F'(s);\ F_2(\bar{x}) = \Phi'(s)\frac{ds}{d\bar{x}}.$

By taking into account the formula (VI.3.12), which connects x with s, equation (X.2.32) can be written in another form:

$$-\frac{\partial \bar{x}^*}{\partial t}\frac{ds}{d\bar{x}} + a^2\frac{\partial}{\partial \bar{x}}\left(F_2(\bar{x})\frac{\partial \bar{x}^*}{\partial \bar{x}}\right) = 0. \qquad (X.2.33)$$

With small t it is possible to search for x * (x, t) in the form:

$$\bar{x}^* = X(\bar{x})\,e^{-\lambda t}. \qquad (X.2.34)$$

Then equation (X.2.33) comes to the following:

$$\lambda\varphi_1(\bar{x}) + a^2 \frac{d}{d\bar{x}}\left(F_2(\bar{x})\frac{dX}{d\bar{x}}\right) = 0. \qquad (X.2.35)$$

In order to investigate stability, it is necessary thus to solve the problem of eigenvalues for equation (X.2.35) under the boundary conditions X = 0 with $\bar{x} = \pm A$. If $\lambda < 0$, current is unstable, if $\lambda > 0$, it is stable.

Page 270.

From the given in chapter VI, §3 description of the distribution of saturation in the stabilized zone, it is not difficult to ascertain that

$$\varphi_1(\bar{x}) = \frac{ds}{d\bar{x}} \leqslant 0 \quad \text{and} \quad F_2(\bar{x}) = \Phi'(s)\frac{ds}{d\bar{x}} \leqslant 0.$$

In accordance with common/general/total theory to the smallest eigenvalue $\lambda$ in stated boundary problem corresponds the eigenfunction, which becomes zero only at the end/leads of the interval in question. Then, integrating equation (X.2.35) for $\overline{x}$ from A to +A, it is easy to obtain that $\lambda_{мин} \geq 0$ with any A. This means that with respect to one-dimensional disturbance/perturbations in any final interval the stabilized zone is always stable. If we examine infinite interval, then $\lambda = 0$ also it can be the eigenvalue to which corresponds eigenfunction $X = X_0$ = const. It is obvious, the disturbance/perturbation of a similar form, i.e., shift/shear along the axis x, it does not disrupt the stability of the stabilized zone (see the analogous problem of the stability of the stabilized zone (see the analogous problem of the stability of flame - G. I. Barenblatt and Ya. B. Zeldovich [19]).

In the process of the displacement of the being mixed liquids stabilizing correction on the motion of interface exerts the mixing of liquids in transition zone, i.e., the dispersion and molecular diffusion, since the mixing leads to the smoothing of the random disturbances of saturation. To consider the effect of mixing on the

st[illegible]lity of interface by a change in the boundary conditions is impossible at least because the width of transition zone with displacement of the being mixed liquids unlimitedly grow/rises in the course of time (is proportional $\sqrt{t}$, cm. §1). Thus, for stability analysis it is necessary to examine complete system of equations for a saturation (X.1.8).

Completely this problem thus far is not solved; however, were obtained several approximate solutions [130, 142], each of which is connected with together of the sufficiently substantially limiting assumptions.

4. In order to describe displacement after loss of stability, it is possible to act by two methods. The first of them entails that in order to trace the formation of the balanced system of the languages, which are formed as a result of a small initial distortion of boundary (for example sinusoidal). The description of the development of language for the case when the viscosity of the displacing liquid is negligible, was conducted with Saffman and by Taylor [155]. Subsequently appeared another a series of the works in which was considered the viscosity of the displacing phase. However, in this manner it is possible to examine the only initial stage of

development of languages both due to the mathematical difficulties and because the correct form of languages cannot be retained unlimitedly for long. The latter is caused by the fact that on the lateral surface of the extended languages appear the secondary distortions, when the length of language exceeds the critical wavelength of disturbance/perturbation.

For the simplified description of the consistent filtration of two liquids during this chaotic motion, let us examine the case of plane flow, when the extent of languages in longitudinal direction is considerably more their width (see Fig. X.3).

Page 271.

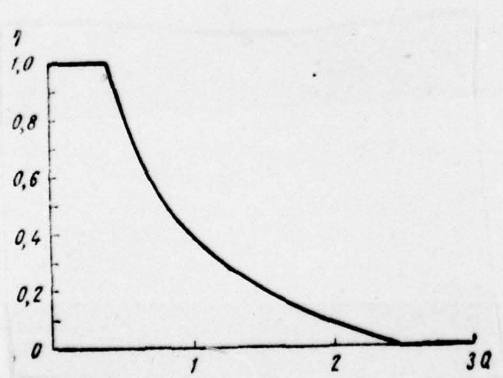


Fig. X.6.

Let us assume that on the average the current is one-dimensional, i.e., the rate of filtration of each of the liquids, averaged along the section, which contains the large number of languages, is directed along X-axis. Let us accept also, that saturation (or concentration) $s_1$ and $s_2$ are constant on both sides of boundary (i.e. in regions 1 and 2). Then, disregarding filtration and exchange in transverse direction, we can write

$$\langle u_i \rangle = -\frac{k_i}{\mu_i} \langle \frac{\partial p}{\partial x} \rangle \frac{h}{H} \qquad (i = 1,\ 2), \qquad \text{(X.2.36)}$$

where h - the total height/altitude of section, occupied with the first (displacing) liquid.

The equations of continuity for the averaged current take the form:

$$H \frac{\partial \langle u_1 \rangle}{\partial x} + m s_1 \frac{\partial h}{\partial t} = 0;$$
$$H \frac{\partial \langle u_2 \rangle}{\partial x} -$$
$$- m (1 - s_2) \frac{\partial h}{\partial t} = 0.$$
$$\text{(X.2.37)}$$

Eliminating from the equations (X.2.37) of $\langle \frac{\partial p}{\partial x} \rangle$ and taking into account that $(1 - s_2)$ the $\langle u_1 \rangle + s_1 \langle u_2 \rangle = u_0$, where $u_0$ - the average rate of filtration, we will obtain one equation for an $\eta = \frac{h}{H}$, analogous with that, as was obtained equation (VI.2.11):

$$\frac{\partial\eta}{\partial t}+\frac{u_0}{m}F'(\eta)\frac{\partial\eta}{\partial x}=0, \qquad (X.2.38)$$

where $F(\eta)=\frac{\eta}{\eta+\mu_*(1-\eta)}$; $\mu_*=\frac{k_2\mu_1 s_1}{k_1\mu_2(1-s_2)}$.

The solution of equation (X.2.38) is obtained exactly as the solutions to the equation of Bakley-Leverett (VI.2.11):

$$x(\eta)=F'(\eta)\int_0^t u_0(\tau)\,d\tau+x_0(\eta)=QF'(\eta)+x_0(\eta). \qquad (X.2.39)$$

. The dependence of the $\eta(Q)$, which corresponds to formula (X.2.39), is represented in Fig. X.6. Since the function of $F'(\eta)$ is monotonic, the jumps of the average saturation cf aaaa in this case are nct formed.

The solution of form (X.2.39) was examined by A. M. Pirverdyan [91] and by I. A. Charn [119] in connection with the formation/education of the language of irrigation in inclined layer and A. Sheydegger [156] for the general case of unstable current. For the displacement of the being mixed liquids, close diagrams were proposed by Perrin [151] and Koval [140].

Page 272.

Application/appendix.

A some formulas of vector analysis.

In the book are used some simplest formulas and the concepts of vector analysis. Are given below the determinations of these values. More detailed information can be found in textbooks.

1. To scalar (numerical function) $\phi$ $(x_1, x_2, x_3)$ it is placed in conformity the vector of the gradient:

$$\operatorname{grad}\varphi\left\{\frac{\partial\varphi}{\partial x_1},\ \frac{\partial\varphi}{\partial x_2},\ \frac{\partial\varphi}{\partial x_3}\right\}. \qquad \text{(A1)}$$

2. Vector a with components $a_1$, $a_2$, $a_3$ corresponds scalar div $\vec{a}$ (divergence $\vec{a}$):

$$\operatorname{div}\vec{a}=\frac{\partial a_1}{\partial x_1}+\frac{\partial a_2}{\partial x_2}+\frac{\partial a_3}{\partial x_3}. \qquad \text{(A2)}$$

Let us designate $\vec{i}_1$, $\vec{i}_2$, $\vec{i}_3$ - single vectors in the direction of axes $x_1$, $x_2$ and $x_3$ and introduce Hamilton's operator (del) $\nabla$:

$$\vec{\nabla}=\vec{i}_1\frac{\partial}{\partial x_1}+\vec{i}_2\frac{\partial}{\partial x_2}+\vec{i}_3\frac{\partial}{\partial x_3}. \qquad (A3)$$

. With the operator of Hamilton it is possible to formally produce actions as with vector, according to the rules of vector algebra. We have in this case

$$\mathrm{grad}\,\varphi=\nabla\varphi; \qquad (A4)$$

$$\mathrm{div}\,\vec{a}=\nabla\vec{a}. \qquad (A5)$$

. Finally,

$$\mathrm{div}\,(\mathrm{grad}\,\varphi)=\nabla(\nabla\varphi)=\nabla^2\varphi=\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}+\frac{\partial^2\varphi}{\partial x_3^2}. \qquad (A6)$$

Operator

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \qquad (A7)$$

is expressed by the operator of Laplace. In cylindrical coordinates (r, $\phi$, z) its expression takes the form:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}. \qquad (A8)$$

. Page 273.

Finally, frequently is utilized the formula of the transformation of surface integral intc volumetric (Ostrogradskiy - Gauss's formula):

$$\iint_S a_n\, dS = \iiint_V \operatorname{div} \vec{a}\, dV. \qquad \text{(A9)}$$

Here $\vec{a}$ is certain vector, assigned in volume by V, surrounded surface S; $a_n$ - the projection of vector $\vec{a}$ at certain point of surface S on

the direction of standard to this surface.

B. Designations of some special functions.

Exponential integral

$$-\mathrm{Ei}(-x) = \int_x^{\infty} e^{-\xi} \frac{d\xi}{\xi}. \tag{B1}$$

Function of errors

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi; \quad \operatorname{erfc} x = 1 - \operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2} d\xi. \tag{B2}$$

Bessel function of the n order of the first and second kind:

$J_n(x)$; $Y_n(x)$.

. the modified Bessel functions of the n order of the first and second kind: $I_n(x)$, $K_n(x)$, the function of $K_n(x)$ it is called also the function of Macdonald.

Bessel functions satisfy the differential equation of Bessel:

$$x^2y''+xy'+(x^2-n^2)\,y=0, \qquad \text{(B3)}$$

a the modified Bessel functions - to this equation, but only with sign (+) before $n^2$. The necessary properties of the enumerated special functions can be found in book [74] or in handbook [26].

C some information from operating calculation.

In the operational calculus of each function f (t) by

alternating/variable t, determined with $0 \leqslant t < \infty$, is placed in conformity the image

$$F(\sigma) = \int_0^\infty e^{-\sigma t} f(t)\, dt \equiv L\{f(t)\}. \tag{C1}$$

. Relationship/ratio (C1) is called of the Laplace transform; variable $\sigma$ - the parameter of the Laplace transform. If we use the Laplace transform to the derivative f' (t) and to fulfill integration in parts, then we will obtain

$$L\{f'(t)\} = \sigma F(\sigma) - f(0). \tag{C2}$$

Specifically, if f (t) = 0 with t = 0, then

$$L\{f'(t)\} = \sigma F(\sigma). \tag{C3}$$

. Thus, to the functional operation of differentiation for functions corresponds the algebraic operation of multiplication by the parameter $\sigma$.

Page 274.

Therefore, by applying the Laplace transform to certain differential equation, which contains differentiation with respect to time, it is possible to arrive at the new equation in which instead of argument t enters the argument $\sigma$ already as the parameter [comp. equations (III.1.1) and (III.1.5) ]. Solving this equation, we find image F ($\sigma$). Are at present vast tables of Laplace transforms, which make it possible in a number of cases to directly find the necessary image and the corresponding to it integral. If this does not succeed in making, then it is often possible to bring together the case in question to tabular (receptions of such an information also are set forth in handbooks). For example let it is necessary to find the original, which corresponds image

$$F(\sigma) = [(\sigma + \alpha)\sigma]^{-1}.$$

Through table we find

$$[\sigma + \alpha]^{-1} \leftarrow\rightarrow e^{-\alpha t}. \qquad \text{(C4)}$$

Taking into account, further, that the multiplication of image on $\sigma$ corresponds to differentiation of original [formula (C2)], we obtain

$$f'(t) = e^{-\alpha t}; \quad f(t) = \int_0^t e^{-\alpha\tau}\, d\tau = \frac{1}{\alpha}[1 - e^{-\alpha t}].$$

The same result can be obtained in another way. Let us present $F(\sigma)$ in the form:

$$F(\sigma)=\frac{1}{\sigma(\sigma+\alpha)}=\frac{1}{\alpha}\left[\frac{1}{\sigma}-\frac{1}{\sigma+\alpha}\right].$$

Hence, utilizing a linearity of Laplace's transformation, according to formula (C4) we obtain expression for f (t).

Finally, in the general case original is found through image through inversion formula:

$$f(t)=\frac{1}{2\pi i}\int\limits_{\gamma-i\infty}^{\gamma+i\infty}F(\sigma)\,e^{\sigma t}\frac{d\sigma}{\sigma}. \qquad \text{(C5)}$$

. Here F ($\sigma$) is considered as complex variable function $\sigma$ = ξ + aaaa, and integral it is undertaken along the direct/straight, parallel imaginary axis of aaaa and that which was arrange/located right it. The inversion formula (C5) is simultaneously and the most complex, and most universal means for the analysis of the solution, obtained by operational calculus. According to the theory of complex variable functions, the way of complex integration can be under specific conditions deformed, without changing the value of integral. This allows in a number of cases either to explicitly calculate integral (C5), or to investigate its properties. Specifically, is established/installed following communication/connection between the

form of the function F $(\sigma)$ and the asymptotic behavior of function f (t) with t $\to\infty$.

Let us examine all the singular points of function F $(\sigma)$ $(\sigma_0, \sigma_1, \sigma_2, \ldots)$, considering them numbered by way of the decrease of real parts (Re $\sigma_0$ > Re $\sigma_1$ > Re $\sigma_2$ > ...). Then occurs theorem [42].

If image F $(\sigma)$ can be decomposed in the vicinity of point $\sigma_0$ in power series

$$F(\sigma)=\sum_{\nu=0}^{\infty} c_\nu (\sigma-\sigma_0)^{\lambda_\nu}, \quad (-N<\lambda_0<\lambda_1<\ldots) \tag{C6}$$

with the arbitrary indices (by not necessarily integral), then original f (t) with t $\to\infty$ can be presented in the form of the asymptotic expansion:

$$f(t) \approx e^{\sigma_0 t} \sum_{\nu=0}^{\infty} \frac{c_\nu}{\Gamma(-\lambda_\nu)} t^{-\lambda_\nu - 1}. \qquad \text{(C7)}$$

in which it is necessary to place $1/\Gamma(-\lambda_\nu) = 0,$ **if** $\lambda_\nu$ takes values 0; 1; 2;: .

Page 275.

Specifically, hence it follows that if the expansion of degree, then relationship/ratio (C7) is converted into the final asymptotic formula. Therefore difference

$$f_1(t) = f(t) - e^{\sigma_0 t} \sum_{\nu=0}^{\infty} \frac{c_\nu}{\Gamma(-\lambda_\nu)} t^{-\lambda_\nu - 1} \qquad \text{(C8)}$$

vanishes faster (or it grow/rises more slowly) than the $e^{(\sigma_0 - \varepsilon) t}$,

where the $\varepsilon$ are a sufficiently small number.

It is possible to show that the asymptotic behavior $f_1$ (t) with $t \to \infty$ is determined by behavior F ($\sigma$) near singular point $\sigma = \sigma_1$ thus, by which the asymptotic behavior f (t) is determined by behavior F ($\sigma$) near point $\sigma = \sigma_0$.

If several singular image points they have identical real parts, the asymptotic behavior of original it turns out to be more complex. The corresponding results are given also in the literature [42].

D. Dimensional analysis and similarity.

In filtration, theory the dimensional analysis and similarity plays significant role. The considerations of the dimensional analysis and similarity are simple, but are not trivial; they are based on several determinations and the facts which it is advisable to present here without proofs.

1. All the physical quantities are expressed by the numbers, which are obtained by means of their comparison with the units of measurement. The unit of measurement are divided into the basic (for example the unit of mass - 1 g, the unit of length - 1 cm. etc.) and derivatives, which are obtained from fundamental units on the basis of the determination of the corresponding values (unit of velocity - 1 cm/s, the unit of force is 1 gcm/s $^{2}$ etc.). The system of the units of measurement is called the totality of the units of measurement, sufficient for characteristic measurement of the class of phenomena in question. For example for the class of mechanical phenomena standard system is the system of SI along with which are applied cgs systems (see g, s), μs (m, kgs (force), s). The class of the systems of the units of measurement is called the totality of the systems of the units of measurement, which differ only in terms of the value of the fundamental units of measurement. For example from the system of SI is obtained the class of systems

$$\frac{\text{[illegible]}}{M};\quad \frac{m}{L};\quad \frac{\text{[illegible]}}{T}, \qquad \text{(D1)}$$

in which the fundamental units of mass, length and time are obtained respectively by decrease in M, by L, T once of the arbitrarily

selected units of mass, length and time: kilogram, meter and seconds. The class of the systems of the units of measurement is designated by the capital letterta of the values, the units of measurement of which are accepted as basic; simultaneously these letters mean, in how often decreases fundamental unit upon transition from one system to another within this class. For example class (1) is designated MLT, by the dimensionality of data are size it is called the expression which it shows, in how often changes the unit of the measurement the datum upon transition from one system to another within this class. It is natural that the dimensionality depends substantially on the class of the systems of the units of measurement, for example in class MLT - the dimensionality of velocity $LT^{1}$, of force $MLT^{2}$ etc.

Page 276.

If the dimensionality of value in this class is identically equal to unity, value is called dimensionless. The dimensionality of certain value f is designated by symbol [f].

2. In the given examples the dimensionality always was represented by exponential monomial. It is possible to show that this

- common/general/total fact, since all systems within this class are equal. Equality of rights means that the dimensionality depends only on that, in how often change the fundamental units of the system of the units of measurement upon transition from one system to another within this class of the systems of the units of measurement, but it does not depend on that, which precisely system of the units of measurement was initial.

3. Values $a_1$, $a_2$ ..., $a_K$ have independent dimensionality, if for one of them it is not possible to present dimensionality in the form of the product of the degrees of the dimensionality of the others. For example density $\rho$, force f and velocity v have the independent dimensionality $ML^{-3}$, $MLT^{-2}$, $LT^{-1}$. Naprotiv, the dimensionality of length , of velocity v and of acceleration w are depended.

4. The physical law can be presented in the form of one or several dependences:

$$a=f(a_1, \ldots, a_k, a_{k+1}, \ldots, a_n.) \qquad \text{(D2)}$$

. Function f depends on n of variables, as which must be selected all values, which determine the characteristics of the phenomenon in question. If is known the mathematical formulation of the problem, then values $a_1, \ldots, a_k, a_{k+1}, \ldots, a_n$ are the independent variables and the parameters, which enter the equations and the supplementary agree (initial and boundary), that determine the unique solution of equation. If the mathematical formulation of the problem is unknown, the selection of values $a_1, \ldots, a_n$ - the question of researcher's intuition.

Let us assume that values $a_1, \ldots, a_k$ have independent dimensionality, and dimensionality of the values of $a_1, \ldots, a_k$ they are expressed as the dimensionality of values $a_1, \ldots,$ into the $a_k$:

$$[a_{k+1}] = [a_1]^{\alpha_1} \ldots [a_k]^{\varkappa_1};$$
$$\ldots\ldots\ldots\ldots\ldots \quad \text{(D3)}$$
$$[a_n] = [a_1]^{\alpha_{n-k}} \ldots [a_k]^{\varkappa_{n-k}}.$$

The dimensionality of value a knowingly is expressed as the dimensionality of values $a_1$, ..., the $a_k$:

$$[a] = [a_1]^{\alpha} \ldots [a_k]^{\varkappa}. \quad \text{(D4)}$$

Otherwise this it would mean that the copy of values $a_1$, ..., $a_n$, determining value, was incomplete: changing the fundamental units of measurement so, in order to value $a_1$, ..., $a_k$, and, consequently, also $a_{k+1}, \ldots, a_n$ would remain constant/invariable, we could change value a, without changing values $a_1$, ..., $a_n$.

Let us introduce values

$$\pi = \frac{a}{a_1^{\alpha} \ldots a_k^{\varkappa}}; \quad \pi_1 = \frac{a_{k+1}}{a_1^{\alpha_1} \ldots a_k^{\varkappa_1}}, \ldots, \pi_{n-k} = \frac{a_n}{a_1^{\alpha_{n-k}} \ldots a_k^{\varkappa_{n-k}}}. \qquad (D5)$$

It is not difficult to check that these values are dimensionless. It is possible to demonstrate the following fact. The dependence (D2), expressed as function n of variables, can be presented through function n - k of variables:

$$\pi = F(\pi_1, \pi_2, \ldots, \pi_{n-k}). \qquad (D6)$$

This dependence is called of P-theorem; P-theorem reflects the independence of the physical laws from the selection of the

fundamental units of measurement. Ode has fundamental value, since it makes it possible to decrease the number of parameters, which determine the unknown characteristics of problem.

Page 277.

5. Fundamental value has also a concept of the similarity of phenomena. Phenomena are called similar, if they differ between themselves only in terms of value of the determining parameters $a_1$, ..., $a_n$ and besides so that values $\pi_1$, ..., $\pi_{n-K}$ for these phenomena are identical.

The importance of the concept of the similarity of phenomena is determined by the following considerations. Let us examine two similar phenomena, one of them conditionally let us name model, another - nature; appropriate the magnitude of values we will designate by the indices of (M) and (H).

On the strength of P-theorem and equality of values $\pi_1$, ..., the $\pi_{n-K}$ of value $\pi$ for these two phenomena are equal to:

$$\pi^{(M)} = \pi^{(H)}. \qquad (D7)$$

whence and from (D5) we have

$$a^{(H)} = a^{(M)} \left(\frac{a_1^{(H)}}{a_1^{(M)}}\right)^{\alpha} \cdot \ldots \cdot \left(\frac{a_k^{(H)}}{a_k^{(M)}}\right)^{x}, \qquad (D8)$$

so that a value of $a^{(H)}$ in nature during the provision for a similarity with simple recalculation it is obtained according to the results of value determination of $a^{(M)}$ on model (in a number of cases cheaper, than simpler manufactured etc.). Values $\pi_1$, ..., the $\pi_{n-k}$ whose equality provides the similarity of phenomena, are called therefore the parameters of similarity, or similarity criteria.

Page 278.

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